Creating 3D Virtual Models of Buildings

You have been given the task of creating a 3D virtual model of a building. Let's think about it first: how do you accurately measure and determine the height of the building? In the real world, architects, engineers, and urban planners rely on mathematical concepts to estimate the dimensions of structures crucial for development and construction projects.

Imagine standing outside the building, equipped with simple tools and mathematical know-how, ready to estimate its height and then create its 3D model. Not only will you gain a deeper understanding of the mathematical concepts involved, but you will also witness firsthand how they are applied in practical, real-world situations.

Most importantly, you will learn to make assumptions and recognise limitations involved in creating the model, merely due to the lack of professional tools or precise measurements. In the future, we will be able to create more refined 3D models!

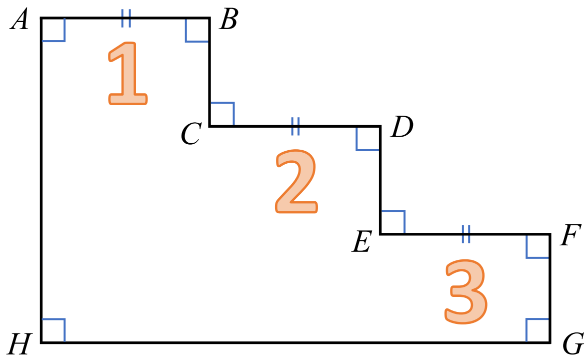
<u>**Creating 3D virtual models of buildings**</u>

Worksheet 1

<u>**Activity 1**</u>

To discuss a mathematical model of height estimation.

1. The following figure shows a podium *ABCDEFGH*.



(a) If $AB = CD = EF = 60$ cm, find the length *HG* of the podium.

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(b) It is known that $BC = 32$ cm. A student estimates the height *AH* as follows.

$$AH = 32\text{ cm} \times 3 = 96\text{ cm}$$

What is the main assumption of his estimation? Is the assumption reasonable?

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2. A student lives in a 17-storey building and wants to estimate its height. He proposes the following approach.

- Step 1: Measure the height of one single step (h) in a flight of staircase.
- Step 2: Count the number of steps (n) in one storey of the building.
- Step 3: Count the number of storeys (k) in the building.
- Step 4: Model formation:

$$\text{The height of the building} = h \cdot n \cdot k$$

On the 12th floor, there are 16 steps in its flight of staircase. He measures the 7th step and finds that it has a riser height of 15.2 cm.
Use his model to estimate the height of the building in m.

3. What are the assumptions and limitations of the model in Question 2?

4. What are some possible situational constraints of using this modelling approach?

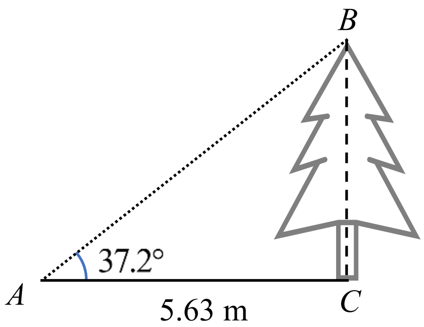
Creating 3D virtual models of buildings

Worksheet 2

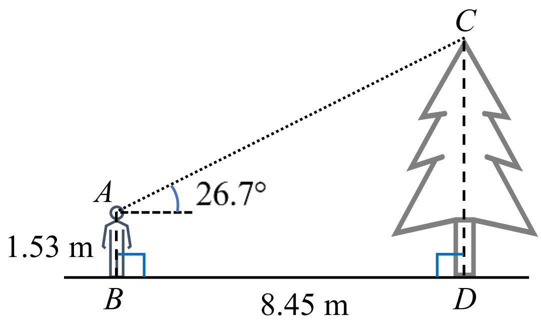
Activity 2A

To estimate heights using trigonometric ratios.

1. In the figure, a tree *BC* casts a shadow 5.63 m long (*AC*) and $\angle A = 37°$. Assume that $BC \perp AC$.
   Find the height of the tree *BC*.



2. In the figure, a student stands at point *B* which is 8.45 m away from a tree *CD*. It is known that his eye level is 1.53 m above the ground. He finds that the angle of elevation of the top of the tree from his eye *A* is $26.7°$.
   Find the height of the tree *CD*.

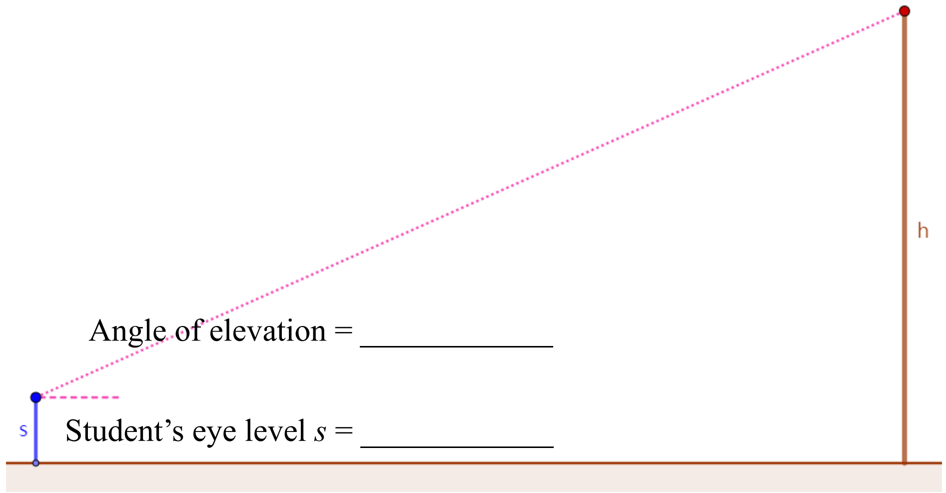


3. Using the method in Question 2, we can estimate the height of a building.

(a) Make the necessary assumptions in the estimation.

(b) Hence, estimate the height h of the building.

[You may refer to the applet: https://www.geogebra.org/m/ymen6puf]



Horizontal distance between the student and building = ____________

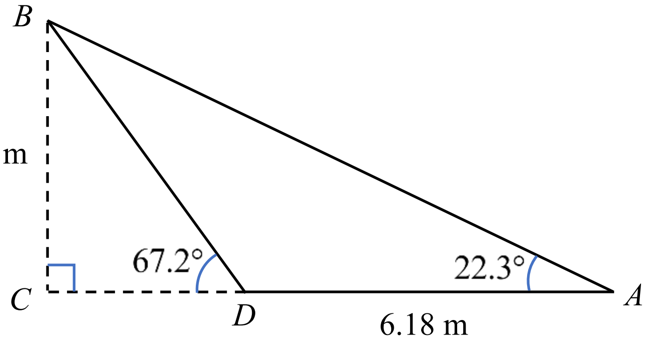
4. What are some possible situational constraints in the above estimation?

Activity 2B

To estimate heights using trigonometric ratios, when measuring horizontal distances is not entirely feasible.

5. Example:

   Given that *CDA* is a straight line. Find the value of *h*.



Solution:

In ΔBCD,

$$\tan 67.2° = \frac{h}{CD}$$

$$CD = \frac{h}{\tan 67.2°} \quad \cdots\cdots(1)$$

In ΔABC,

$$\tan 22.3° = \frac{h}{CD + 6.18}$$

$$CD + 6.18 = \frac{h}{\tan 22.3°} \quad \cdots\cdots(2)$$

By substituting (1) into (2), we have

$$\frac{h}{\tan 67.2°} + 6.18 = \frac{h}{\tan 22.3°}$$

$$6.18 = \frac{h}{\tan 22.3°} - \frac{h}{\tan 67.2°}$$

$$6.18 = h(\frac{1}{\tan 22.3°} - \frac{1}{\tan 67.2°})$$

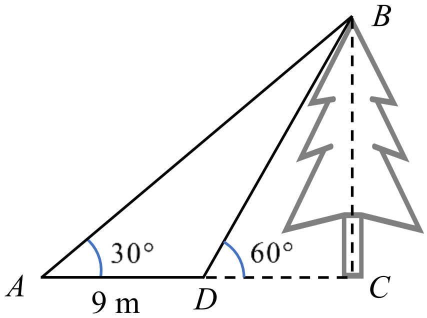
$$6.18 \div (\frac{1}{\tan 22.3°} - \frac{1}{\tan 67.2°}) = h$$

$$h = 3.06$$

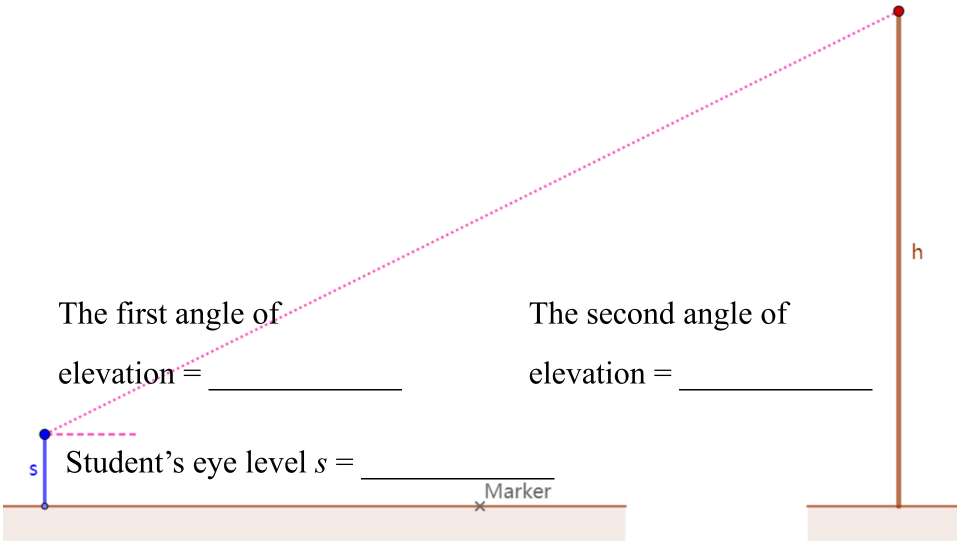
6. Quick Practice:

In the figure, we want to estimate the height of a tree BC. It is known that $\angle A = 30°$, $\angle BDC = 60°$, and the distance between A and D is 9 m. Assume that ADC is a straight line and $BC \perp AC$.

Find the height of the tree BC.



7. Using the method in Question 6, we can estimate the height of the building.
   (a) Make the necessary assumptions in the estimation.
   (b) Hence, estimate the height h of the building.
   [You may refer to the applet: https://www.geogebra.org/m/djurma4f]



The first angle of elevation = ____________

The second angle of elevation = ____________

Student's eye level s = ____________

Horizontal distance between the first and second positions = ____________

Creating 3D virtual models of buildings

Worksheet 3

Activity 3

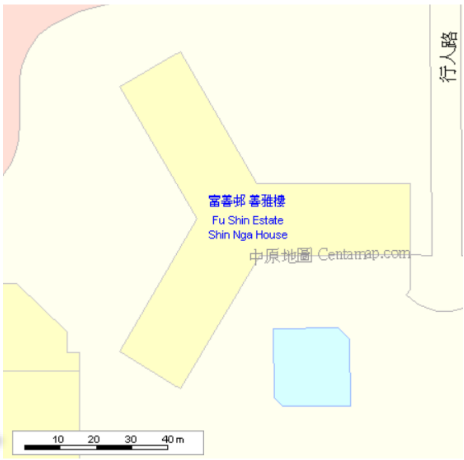
To create a virtual 3D model of a building.

1. After estimating the height of a building, we can use a map to find its lateral dimension.

   There are some online maps in Hong Kong. For example:

   ➢ Centamap.com: http://hk.centamap.com/gc/home.aspx

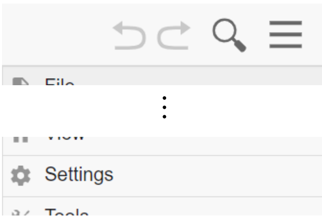
   Screen-capturing or downloading the map of the target building alongside the scale will enable us to create its 3D model based on the scale.



   With the height and lateral dimension of the building, we can create a virtual 3D model using GeoGebra.

2. Go to GeoGebra official website: https://www.geogebra.org/classic

   We can set the language of GeoGebra:



   Main manual (top right-hand corner)

   Settings / 設定 / 设置

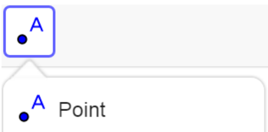
   → Select your preferred language

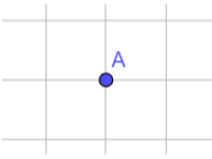
Tips

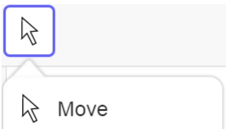
In GeoGebra, after using a tool, it is a good practice to select "Move" tool.

You can understand this practice as:

Pick up a tool → Use the tool → Put down the tool



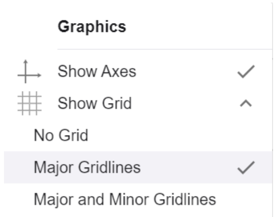




3. The following steps will guide us through creating a 3D model of a building.

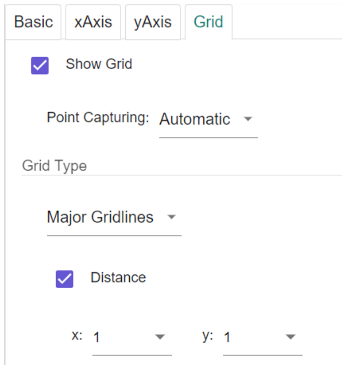
<table>
<tr><th>Step</th><th>Description</th></tr>
<tr><td>i.</td><td>Set the gridlines

- Right-click on the graphics view
- “Show Grid” → Tick “Major Gridlines”



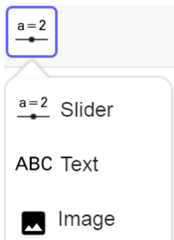
</td></tr>
<tr><td>ii.</td><td>Set the distance of the grid

- Right-click on the graphics view
- Click “Graphics”
- Go to “Grid” tab
→ “Grid Type” → Select “Major Gridlines”
- Tick “Distance”
- Set x = 1 and y = 1

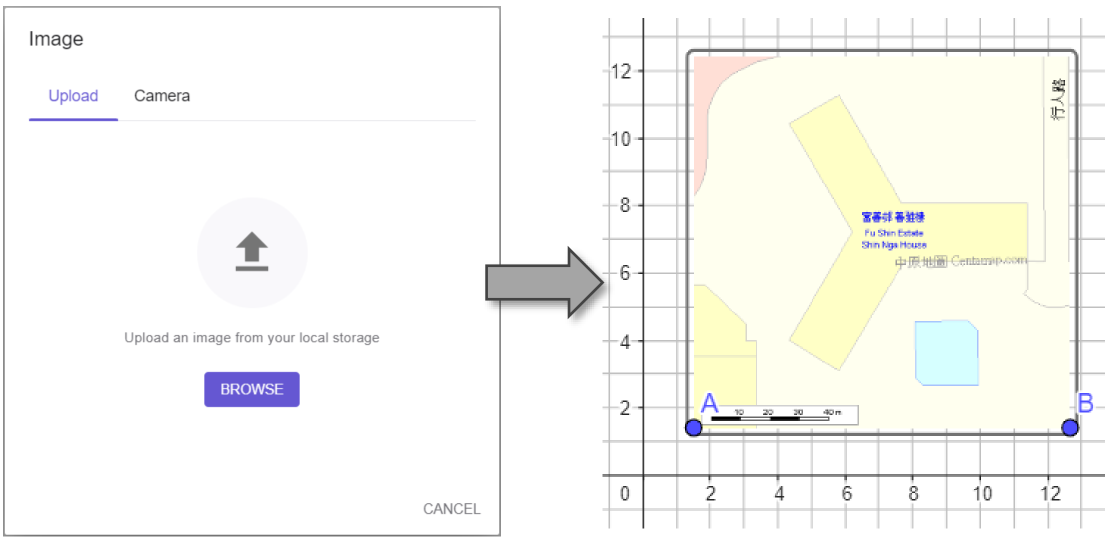


Remark:
This will be useful for you to define the scale of your 3D model.
</td></tr>
<tr><td>iii.</td><td>Import the map of the building

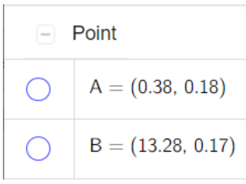
- Use “Image” tool to select an image from files
- Go to “Upload” tab → “BROWSE”
- Upload the map



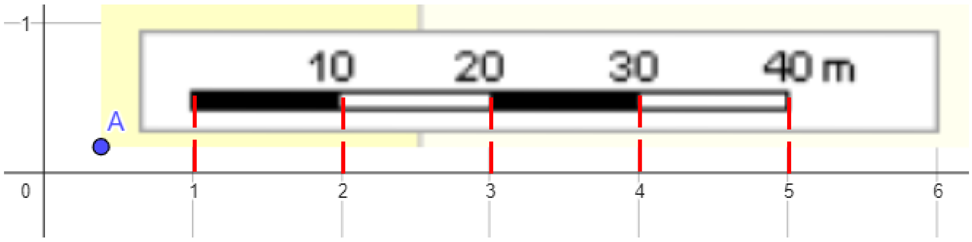
</td></tr>
</table>



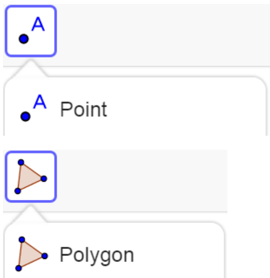
Step	Description
iv.	Adjust the size of the map • Move Points A and B to adjust the size of the map • Hide Points A and B

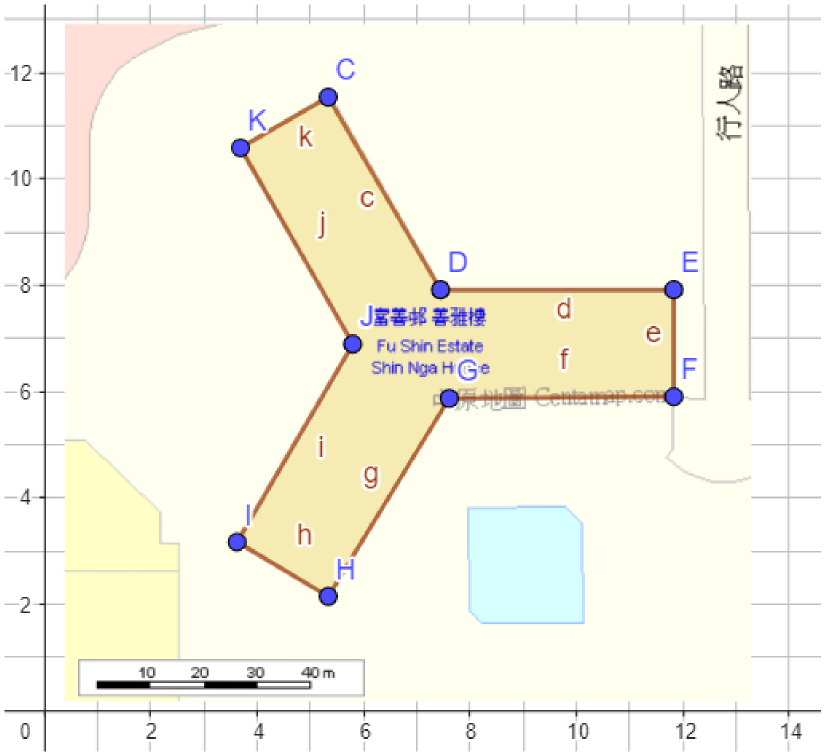


Remark:
If the interval of the gridline matches with the scale of the map, it will be easier to get the proportions right when creating your 3D model.

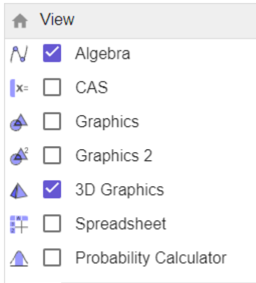


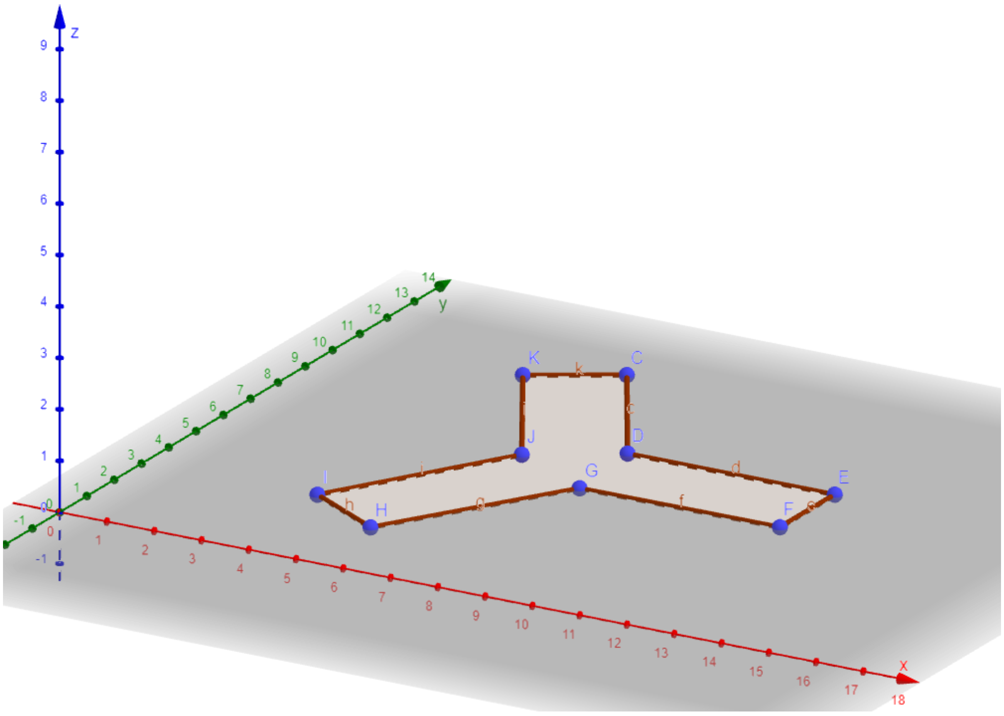
Step	Description
v.	Draw the base of the building • Use "Point" tool to draw the vertices of the building • Use "Polygon" tool to connect all vertices to draw the base of the building





Step	Description
vi.	Show the 3D Graphics view • "Main manual" (top right-hand corner) → "View" • Tick "3D Graphics" view • Untick "Graphics" view • Adjust the view

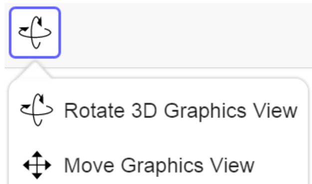




Tips

To adjust the 3D Graphics view,

- Use "Move Graphics View" tool

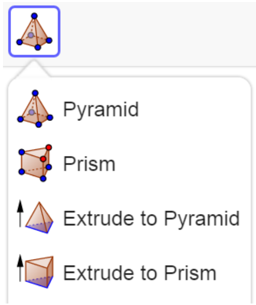
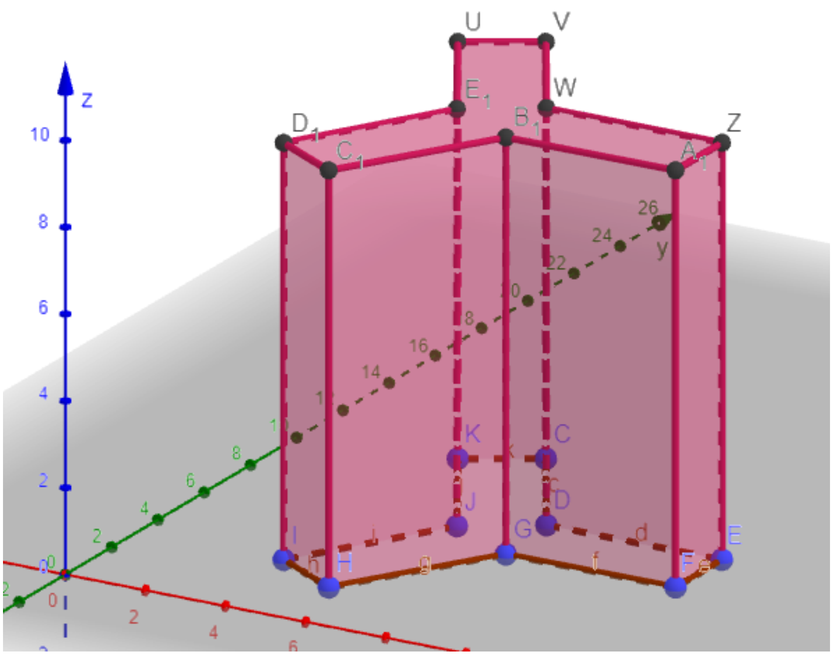


- First, you can move it vertically 
- After one click, you can move it horizontally 
- There are some useful buttons at the bottom right-hand corner

 Standard View: Go back to the initial position and view

 Zoom In

 Zoom Out

Step	Description
vii.	Create the building *(taking a 96-metre-tall building as an example, according to the scale of the map, we need to input 9.6)* • Use “Extrude to Prism” tool and click the base of the building • Input 9.6 as its Altitude   

4. What are the assumptions of this 3D model?

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Imagine standing outside the building, equipped with simple tools and mathematical know-how, ready to estimate its height and then create its 3D model. Not only will you gain a deeper understanding of the mathematical concepts involved, but you will also witness firsthand how they are applied in practical, real-world situations.

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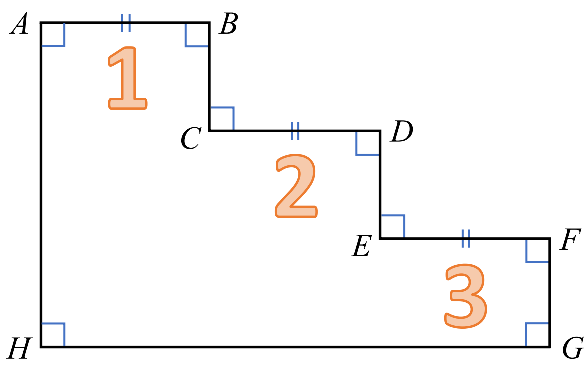
Worksheet 1

Activity 1

To discuss a mathematical model of height estimation.

1. The following figure shows a podium *ABCDEFGH*.



(a) If $AB = CD = EF = 60$ cm, find the length *HG* of the podium.

$GH = 60 \text{ cm} \times 3$

$= 180 \text{ cm}$

(b) It is known that $BC = 32$ cm. A student estimates the height *AH* as follows.

$$AH = 32 \text{ cm} \times 3 = 96 \text{ cm}$$

What is the main assumption of his estimation? Is the assumption reasonable?

The student assumes that each step is of equal height (32 cm).

The assumption is reasonable because of design consistency. In many structures, stairs are often designed to have equal step heights. This design consistency helps ensure safety and ease of use, as people tend to expect uniformity in step heights when climbing stairs.

[The assumption is not reasonable because of possible variations in real-world scenarios. Variations can occur due to factors such as customised design, construction errors, or wear and tear over time.]

2. A student lives in a 17-storey building and wants to estimate its height. He proposes the following approach.

- Step 1: Measure the height of one single step (h) in a flight of staircase.
- Step 2: Count the number of steps (n) in one storey of the building.
- Step 3: Count the number of storeys (k) in the building.
- Step 4: Model formation:

$$\text{The height of the building} = h \cdot n \cdot k$$

On the 12th floor, there are 16 steps in its flight of staircase. He measures the 7th step and finds that it has a riser height of 15.2 cm.
Use his model to estimate the height of the building in m.

The height of the building
$= 15.2 \text{ cm} \times 16 \times 17$
$= 4134.4 \text{ cm}$
$= 41 \text{ m}$

3. What are the assumptions and limitations of the model in Question 2?

Assumptions:
1. Uniform step heights (h)
2. Perpendicular staircase
3. Same number of steps (n) for each storey (k)

Limitations: The model overlooks the sections extending beyond the staircase, such as the rooftop or other architectural components.

4. What are some possible situational constraints of using this modelling approach?

1. Inaccessible areas: In some cases, certain sections of the building may be private areas and inaccessible for step counting and measurement.
2. Safety concern: There may be a potential risk associated with entering building staircases, especially in unfamiliar or unsecured environments.

<u>Creating 3D virtual models of buildings</u>

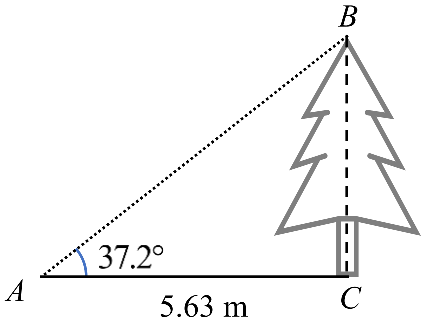
Worksheet 2

<u>Activity 2A</u>

To estimate heights using trigonometric ratios.

1. In the figure, a tree *BC* casts a shadow 5.63 m long (*AC*) and $\angle A = 37°$. Assume that $BC \perp AC$.

   Find the height of the tree *BC*.



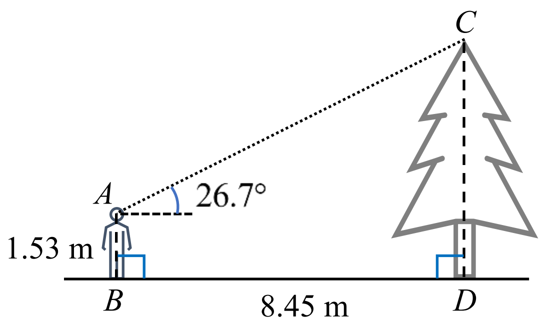
$$\tan 37.2° = \frac{BC}{5.63}$$

$$BC = 5.63 \tan 37.2°$$

$$= 4.27 \text{ m}$$

2. In the figure, a student stands at point *B* which is 8.45 m away from a tree *CD*. It is known that his eye level is 1.53 m above the ground. He finds that the angle of elevation of the top of the tree from his eye *A* is $26.7°$.

   Find the height of the tree *CD*.



Draw $AE \perp CD$

$AE = 8.45 \text{ m}$

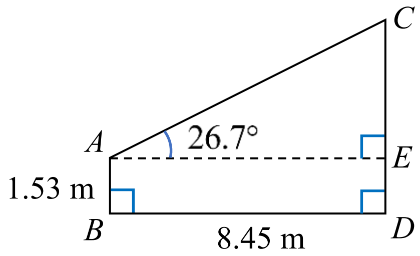
$$\tan 26.7° = \frac{CE}{8.45}$$

$$CE = 8.45 \tan 26.7°$$

$$CD = CE + ED$$

$$= 8.45 \tan 26.7° + 1.53$$

$$= 5.78 \text{ m}$$

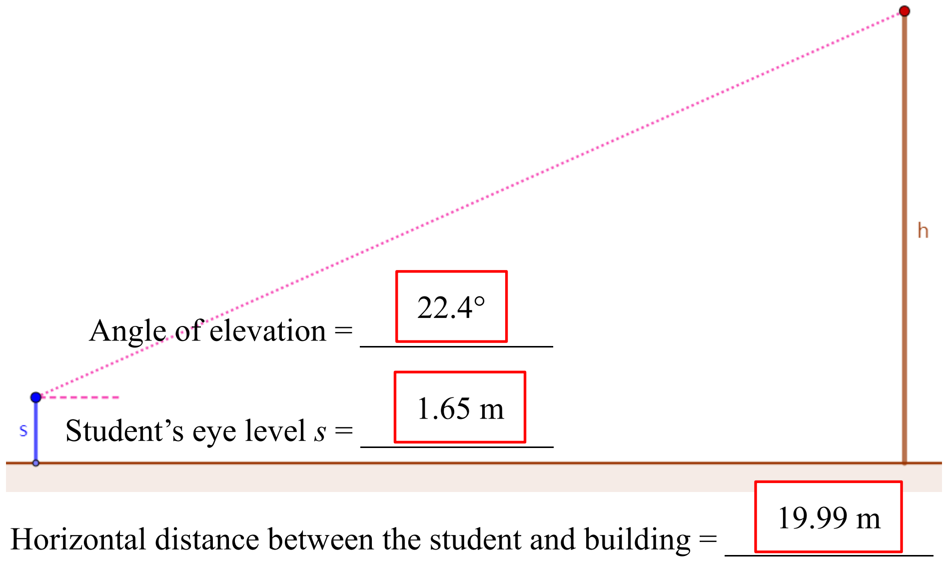


3. Using the method in Question 2, we can estimate the height of a building.

(a) Make the necessary assumptions in the estimation.

(b) Hence, estimate the height h of the building.

[You may refer to the applet: https://www.geogebra.org/m/ymen6puf]



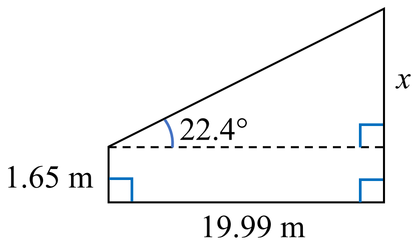
(a) Assumptions:

1. Vertical building: The angle between the ground and the building is exactly 90°.
2. Ground conditions: The ground is free of irregularities, as they can affect the accuracy when measuring the horizontal distance between the student and building.
3. Ground level: The student's standing position and the base of the building are at the same horizontal level.

(b) *[Based on the above set of possible measured values]*

From the student's eye, draw a horizontal line perpendicular to the building.

Let x be the unknown, as shown in the



$$\tan 22.4° = \frac{x}{19.99}$$

$$CE = 19.99 \tan 22.4°$$

$$\begin{aligned} h &= x + 1.65 \\ &= 19.99 \tan 22.4° + 1.65 \\ &= 9.89 \text{ m} \end{aligned}$$

∴ The height of the building is 9.89 m.

4. What are some possible situational constraints in the above estimation?

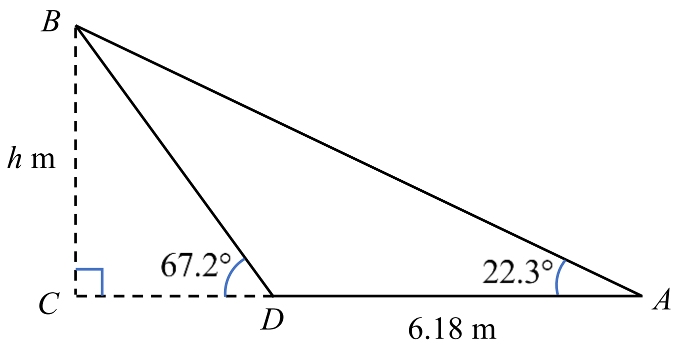
1. Weather conditions: Bad weather conditions (e.g., heavy fog or rain) can obscure the view and affect the accuracy of the angle measurement.
2. Obstructions: There may be obstacles which block the way of measuring the distance between the student's standing position and the base of the building.

Activity 2B

To estimate heights using trigonometric ratios, when measuring horizontal distances is not entirely feasible.

5. Example:

   Given that *CDA* is a straight line. Find the value of *h*.



Solution:

In ΔBCD,

$$\tan 67.2° = \frac{h}{CD}$$

$$CD = \frac{h}{\tan 67.2°} \quad \cdots\cdots(1)$$

In ΔABC,

$$\tan 22.3° = \frac{h}{CD + 6.18}$$

$$CD + 6.18 = \frac{h}{\tan 22.3°} \quad \cdots\cdots(2)$$

By substituting (1) into (2), we have

$$\frac{h}{\tan 67.2°} + 6.18 = \frac{h}{\tan 22.3°}$$

$$6.18 = \frac{h}{\tan 22.3°} - \frac{h}{\tan 67.2°}$$

$$6.18 = h\left(\frac{1}{\tan 22.3°} - \frac{1}{\tan 67.2°}\right)$$

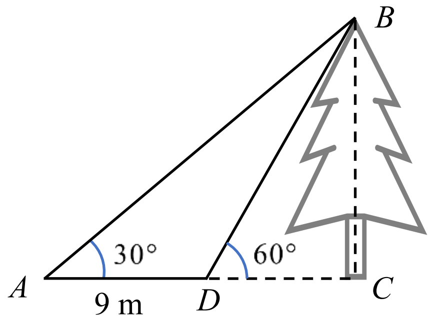
$$6.18 \div \left(\frac{1}{\tan 22.3°} - \frac{1}{\tan 67.2°}\right) = h$$

$$h = 3.06$$

6. <u>Quick Practice:</u>

In the figure, we want to estimate the height of a tree *BC*. It is known that $\angle A = 30°$, $\angle BDC = 60°$, and the distance between *A* and *D* is 9 m. Assume that *ADC* is a straight line and $BC \perp AC$.

Find the height of the tree *BC*.



Let $BC = h$ m and $DC = x$ m.

In ΔBCD,

$$\tan 60° = \frac{h}{x}$$

$$x = \frac{h}{\tan 60°} \quad \cdots\cdots(1)$$

In ΔABC,

$$\tan 30° = \frac{h}{x+9}$$

$$x + 9 = \frac{h}{\tan 30°} \quad \cdots\cdots(2)$$

By substituting (1) into (2), we have

$$\frac{h}{\tan 60°} + 9 = \frac{h}{\tan 30°}$$

$$9 = \frac{h}{\tan 30°} - \frac{h}{\tan 60°}$$

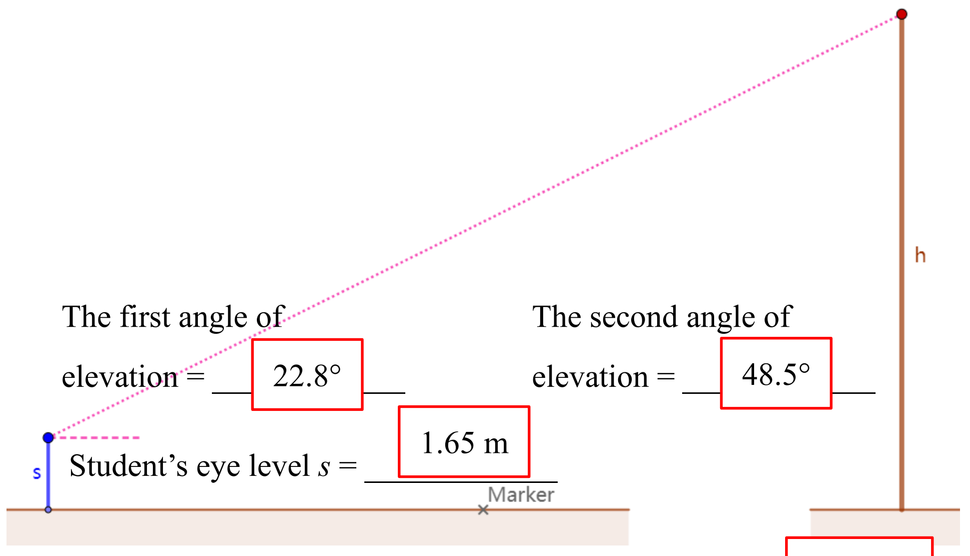
$$9 = h\left(\frac{1}{\tan 30°} - \frac{1}{\tan 60°}\right)$$

$$9 \div \left(\frac{1}{\tan 30°} - \frac{1}{\tan 60°}\right) = h$$

$$h = 7.79$$

$\therefore \; BC = 7.79$ m

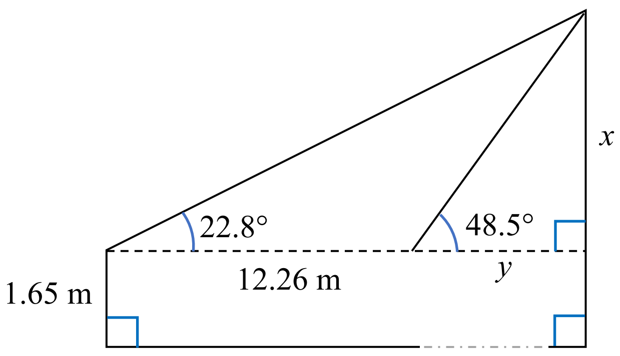
7. Using the method in Question 6, we can estimate the height of the building.
   (a) Make the necessary assumptions in the estimation.
   (b) Hence, estimate the height h of the building.
   [You may refer to the applet: https://www.geogebra.org/m/djurma4f]



Horizontal distance between the first and second positions = 12.26 m

(a) Assumptions:
Ground level: The student's two standing positions and the base of the building are at the same horizontal level and colinear.

(b) *[Based on the above set of possible measured values]*
From the student's eye, draw a horizontal line perpendicular to the building.
Let x and y be the unknowns, as shown in the figure.



$$\tan 48.5° = \frac{x}{y}$$

$$y = \frac{x}{\tan 48.5°} \quad \cdots\cdots(1)$$

$$\tan 22.8° = \frac{x}{y+12.26}$$

$$y+12.26 = \frac{x}{\tan 22.8°} \quad \cdots\cdots(2)$$

By substituting (1) into (2), we have

$$\frac{x}{\tan 48.5°} + 12.26 = \frac{x}{\tan 22.8°}$$

$$12.26 = \frac{x}{\tan 22.8°} - \frac{x}{\tan 48.5°}$$

$$12.26 = x(\frac{1}{\tan 22.8°} - \frac{1}{\tan 48.5°})$$

$$12.26 \div (\frac{1}{\tan 22.8°} - \frac{1}{\tan 48.5°}) = x$$

$$x = 8.205165382$$

$$h = 8.205165382 + 1.65$$

$$= 9.86$$

$\therefore$ The height of the building is 9.86 m.

<u>Creating 3D virtual models of buildings</u>

Worksheet 3

<u>Activity 3</u>

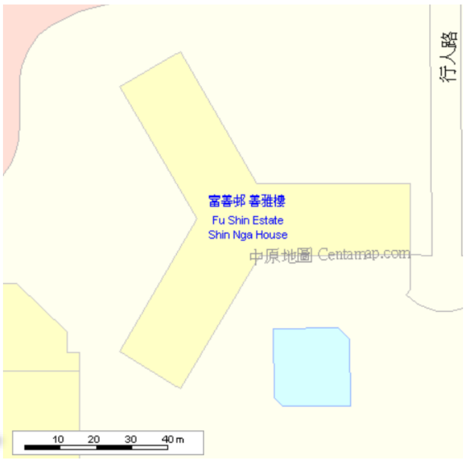
To create a virtual 3D model of a building.

1. After estimating the height of a building, we can use a map to find its lateral dimension.

   There are some online maps in Hong Kong. For example:

   ➢ Centamap.com: http://hk.centamap.com/gc/home.aspx

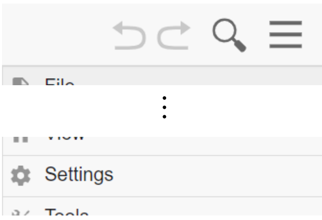
   Screen-capturing or downloading the map of the target building alongside the scale will enable us to create its 3D model based on the scale.



   With the height and lateral dimension of the building, we can create a virtual 3D model using GeoGebra.

2. Go to GeoGebra official website: https://www.geogebra.org/classic

   We can set the language of GeoGebra:



   Main manual (top right-hand corner)

   Settings / 設定 / 设置

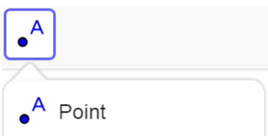
   → Select your preferred language

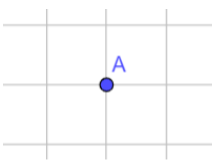
<u>Tips</u>

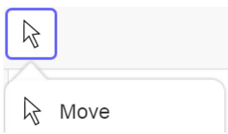
In GeoGebra, after using a tool, it is a good practice to select "Move" tool.

You can understand this practice as:

Pick up a tool → Use the tool → Put down the tool







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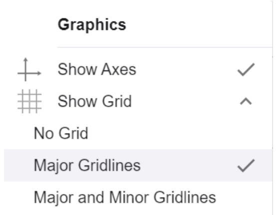
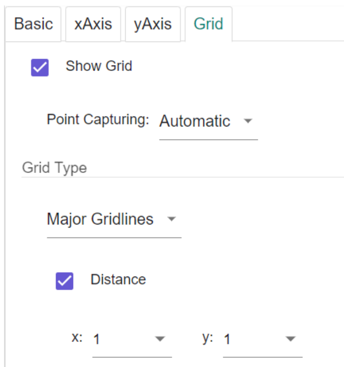
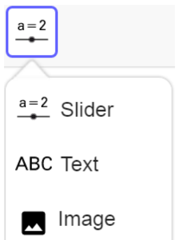
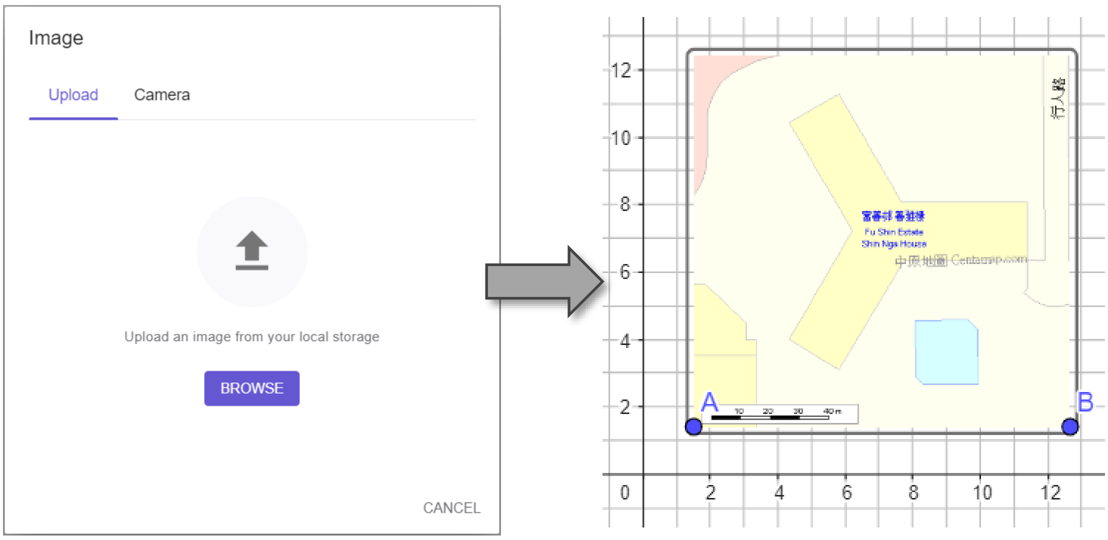
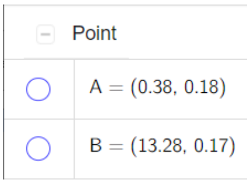
3. The following steps will guide us through creating a 3D model of a building.

<table>
<tr><th>Step</th><th>Description</th></tr>
<tr><td>i.</td><td>Set the gridlines
• Right-click on the graphics view
• “Show Grid” → Tick “Major Gridlines”

</td></tr>
<tr><td>ii.</td><td>Set the distance of the grid
• Right-click on the graphics view
• Click “Graphics”
• Go to “Grid” tab
→ “Grid Type” → Select “Major Gridlines”
• Tick “Distance”
• Set x = 1 and y = 1


Remark:
This will be useful for you to define the scale of your 3D model.</td></tr>
<tr><td>iii.</td><td>Import the map of the building
• Use “Image” tool to select an image from files
• Go to “Upload” tab → “BROWSE”
• Upload the map

</td></tr>
</table>

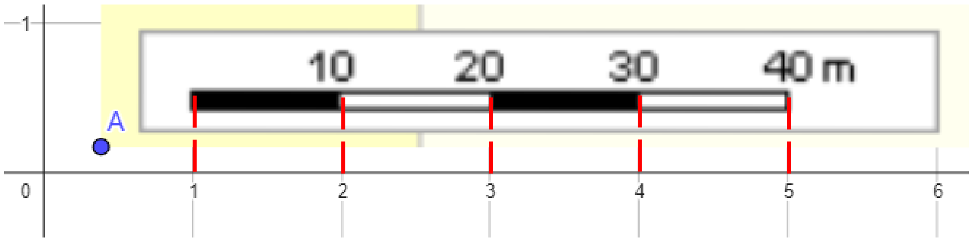


Step	Description
iv.	Adjust the size of the map • Move Points A and B to adjust the size of the map • Hide Points A and B

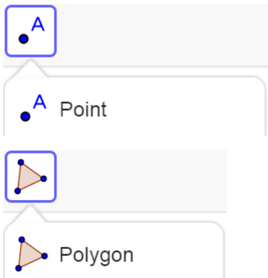


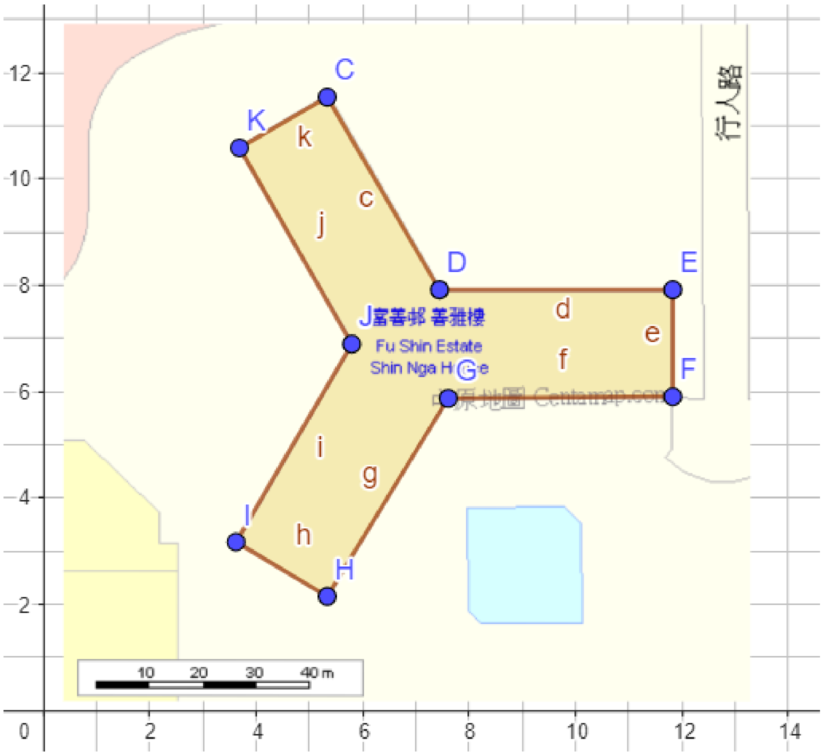
Remark:

If the interval of the gridline matches with the scale of the map, it will be easier to get the proportions right when creating your 3D model.



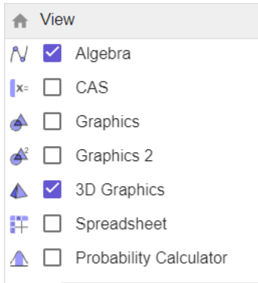
v.	Draw the base of the building • Use “Point” tool to draw the vertices of the building • Use “Polygon” tool to connect all vertices to draw the base of the building

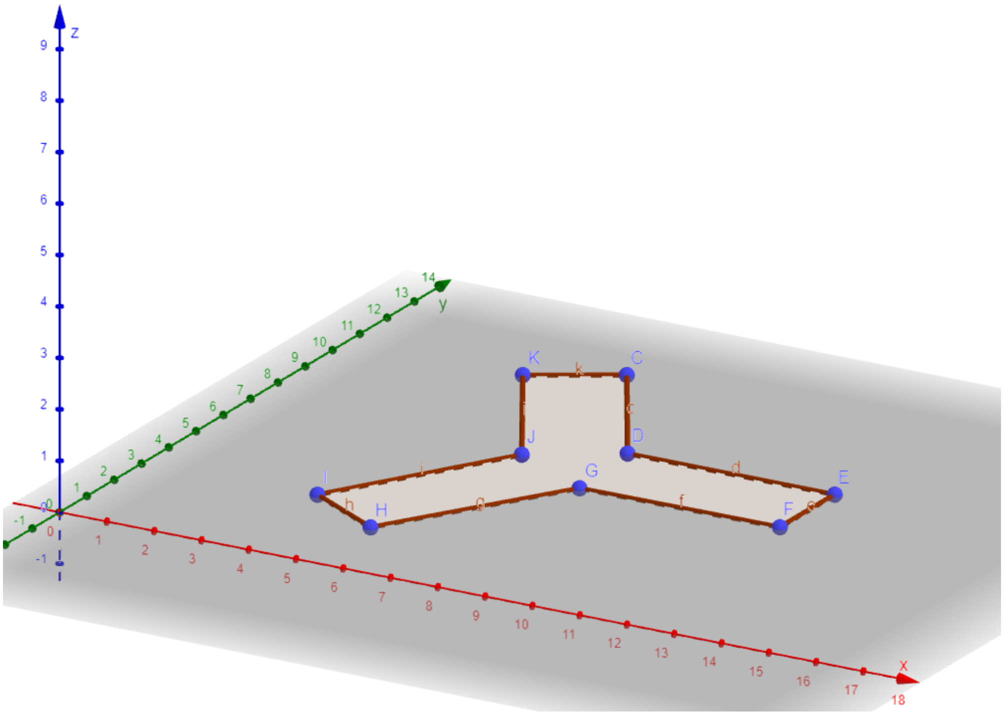




Step	Description
vi.	Show the 3D Graphics view

- "Main manual" (top right-hand corner) → "View"
- Tick "3D Graphics" view
- Untick "Graphics" view
- Adjust the view

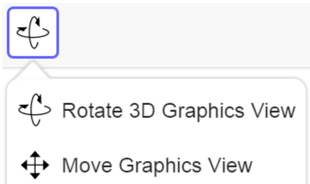




Tips

To adjust the 3D Graphics view,

- Use "Move Graphics View" tool



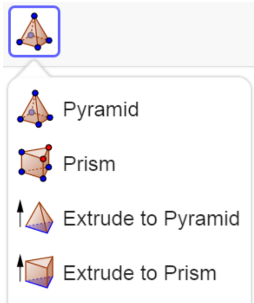
- First, you can move it vertically 
- After one click, you can move it horizontally 
- There are some useful buttons at the bottom right-hand corner

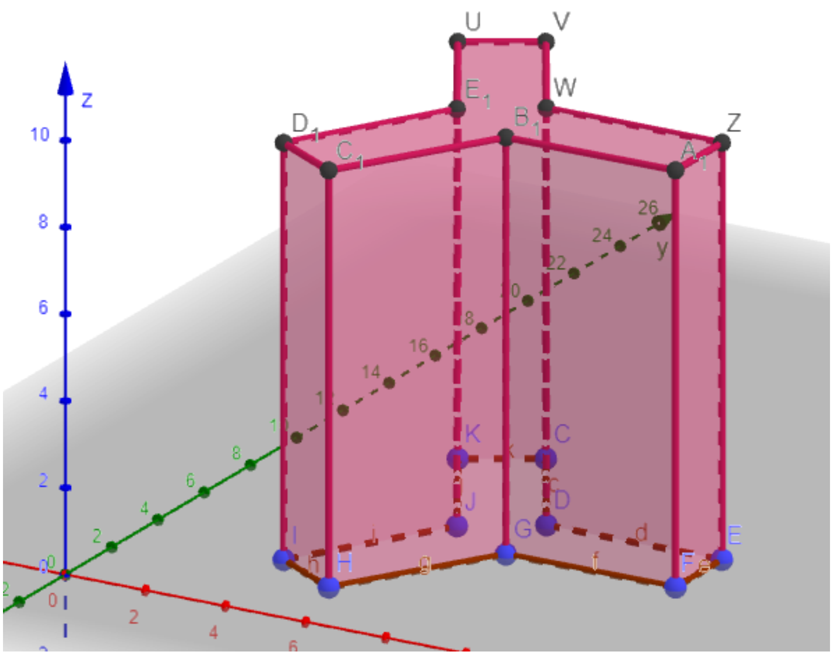
 Standard View: Go back to the initial position and view

 Zoom In

 Zoom Out

Step	Description
vii.	Create the building *(taking a 96-metre-tall building as an example, according to the scale of the map, we need to input 9.6)* • Use “Extrude to Prism” tool and click the base of the building • Input 9.6 as its Altitude





4. What are the assumptions of this 3D model?

1. Building shape: The building is a prism, having a uniform cross-section along its entire height.
2. The top of the building: The roof or uppermost floor of the building is perfectly level.