



## Finding the Optimal Locations for Company Headquarters and Warehouse

Imagine you are the CEO of a company, tasked with finding the perfect locations for your new headquarters and warehouse. This task goes beyond simply picking spots on a map. It involves using mathematical modelling to make informed, strategic decisions that save time and transportation costs.

Consider the scenario in which your headquarters needs to be at the same shortest distance from two key stores. This is a manageable challenge, but it becomes more complicated when you have to consider three or more stores. For the warehouse, the challenge is to position it so that it is equidistant from major roads, optimising transportation routes and delivery times.

In this activity, you will learn to tackle these real-world problems. You will also identify and manage the constraints involved in such decisions. Let's embark on this quest to find the optimal locations for your company's headquarters and warehouse!

## Revision

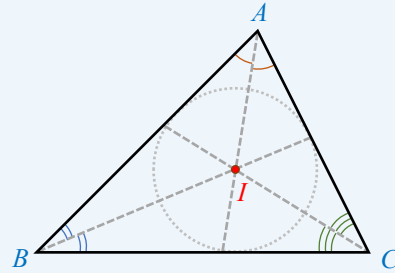
Fill in the blanks with the words provided. They are four special lines in a triangle.

Perpendicular bisectors	Altitudes	Medians	Angle bisectors
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Refer to: <https://www.geogebra.org/m/btwzyahk>

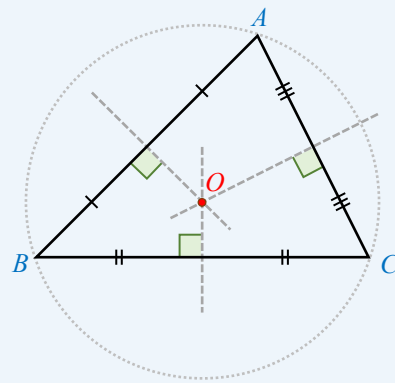
1. Incentre  $I$  is  
the point of intersection of  
the three \_\_\_\_\_  
in a triangle.

*Note:*  $I$  is the centre of the largest circle that can be drawn in the triangle. The circle drawn is called the **inscribed circle**.

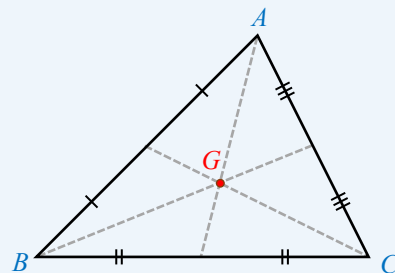


2. Circumcentre  $O$  is  
the point of intersection of  
the three \_\_\_\_\_  
in a triangle.

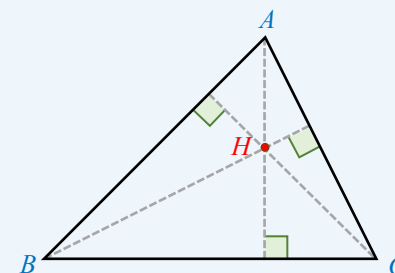
*Note:*  $O$  is the centre of the circle which passes through the three vertices of the triangle. The circle drawn is called the **circumscribed circle**.



3. Centroid  $G$  is  
the point of intersection of  
the three \_\_\_\_\_  
in a triangle.



4. Orthocentre  $H$  is  
the point of intersection of  
the three \_\_\_\_\_  
in a triangle.



## Finding the optimal locations for company headquarters and warehouse

### Worksheet 1

#### Activity 1A

To ensure the same and shortest distance from the headquarters to two stores.

1. Headquarters  $Q$  is the main offices of our company, from where information is delivered to stores via radio technology. If there are two stores  $A$  and  $B$ , our boss wants to ensure that the distances between the headquarters and each of them are the same and the shortest.

Use mathematical terms to describe the requirement.

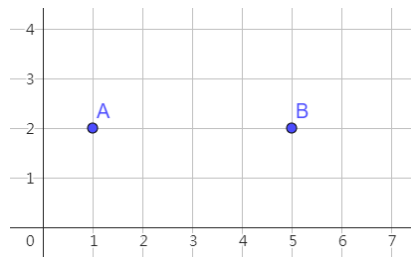
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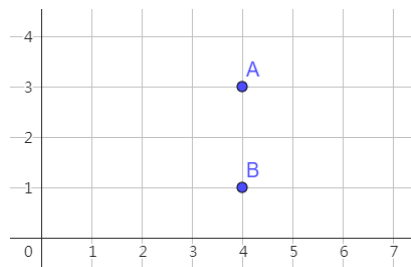
2. The following figures show some specific scenarios. Points  $A$  and  $B$  represent the locations of two stores, respectively. Mark the optimal locations of the headquarters  $Q$  on the figures and write down their coordinates.

(a)



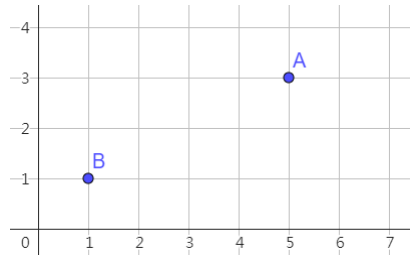
$$Q = ( \quad , \quad )$$

(b)



$$Q = ( \quad , \quad )$$

(c)



$$Q = ( \quad , \quad )$$

3. In reality, store locations vary across different regions.

Let the coordinates of two stores be  $(x_1, y_1)$  and  $(x_2, y_2)$ , respectively.

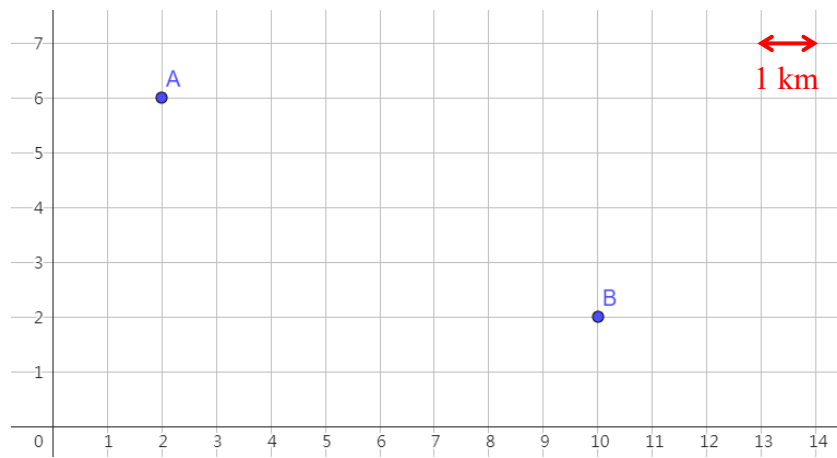
Write down a model of finding the optimal location for the headquarters  $Q$ .

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4. In the following figure, points  $A$  and  $B$  represent the locations of two stores, respectively.



(a) Using the model in Question 3, find the coordinates of the headquarters  $Q$  and mark its location on the figure.

(b) Calculate the distance from our headquarters to each of the stores correct to the nearest 0.01 km.

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5. What assumptions are made in your model in Question 3?

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6. Apart from the same and shortest distance from the headquarters to our stores, what factors should we consider when finding the optimal location for the headquarters?

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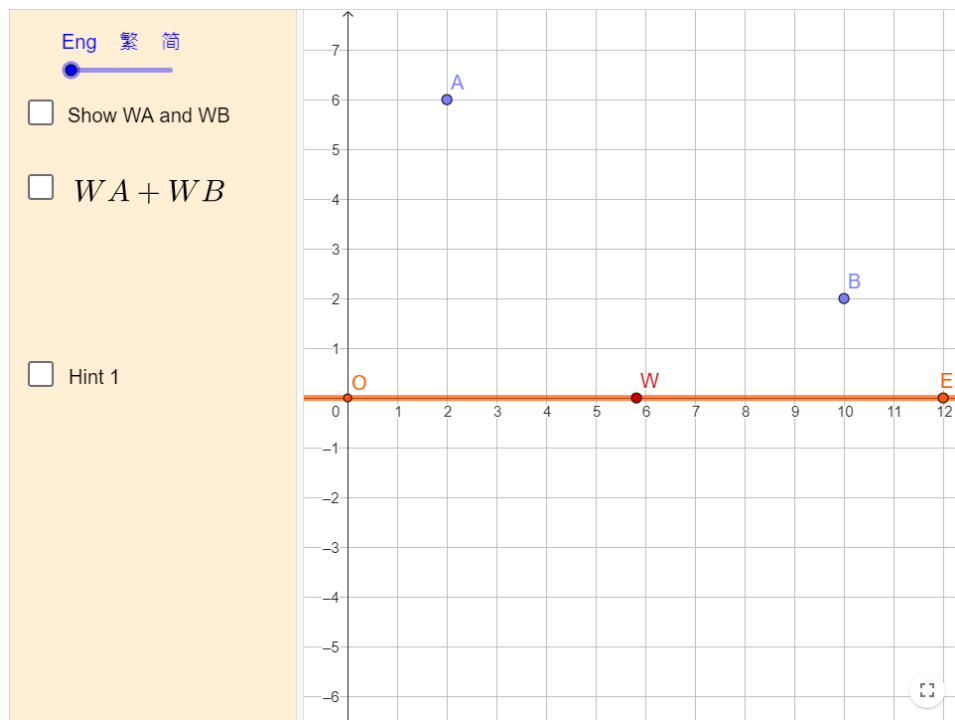
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### Activity 1B

To find the shortest distance between a warehouse and two stores.

7. Warehouse  $W$  is a large building for storing and distributing products to our stores. Our boss is searching for its best location. There are two requirements regarding the selected location:
  - i. To better connect with the rapid transit system,  $W$  must be located beside the distributor road  $OE$ .
  - ii. The total distance between  $W$  and our two stores  $A$  and  $B$  must be minimised. Nevertheless, the distances of paths  $WA$  and  $WB$  need not be the same.

Explore using the following applet: <https://www.geogebra.org/m/a6b9jb2g>



Based on the requirements,  $WA + WB$  should be \_\_\_\_\_.

8. Describe how to find the location of the warehouse  $W$ .  
In the above figure, sketch your steps and mark the location of  $W$ .

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9. What are some possible constraints when searching for the location of the warehouse? How can we compromise?

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## Finding the optimal locations for company headquarters and warehouse

### Worksheet 2

#### Activity 2A

To ensure the same distance from the headquarters to three stores.

- If there are three different stores  $A$ ,  $B$  and  $C$ , our boss wants to ensure that the distances between the headquarters  $Q$  and each of them are the same.

Write down the mathematical representation of this requirement.

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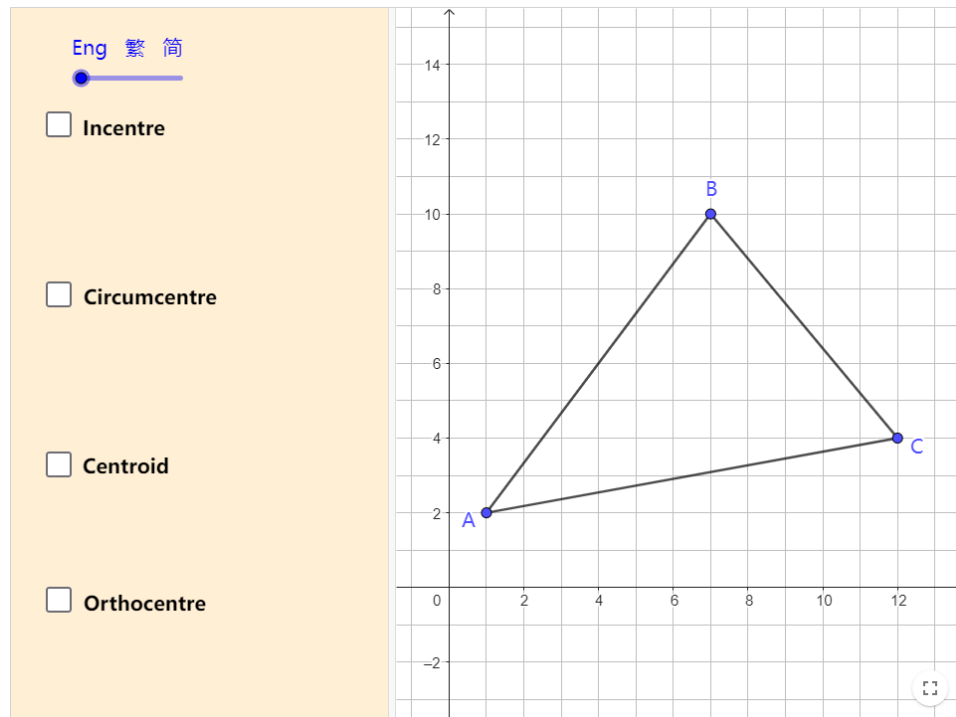


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- Which of the centres of  $\triangle ABC$  can meet our boss's requirement?

Use the following applet to verify your answer.

Link: <https://www.geogebra.org/m/gm6ayhap>



Your choice	Centres	Distance (correct to the nearest 0.01 km)		
		$QA$	$QB$	$QC$
<input checked="" type="checkbox"/>				
	Incentre			
	Circumcentre			
	Centroid			
	Orthocentre			



3. What assumptions are made in finding the location?

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4. Can the selected location minimise the total distance from our headquarters to the three stores?

You can use the following table to help explain your answer.

Centres	Distance (correct to the nearest 0.01 km)			
	<i>QA</i>	<i>QB</i>	<i>QC</i>	Total
Incentre				
Circumcentre				
Centroid				
Orthocentre				

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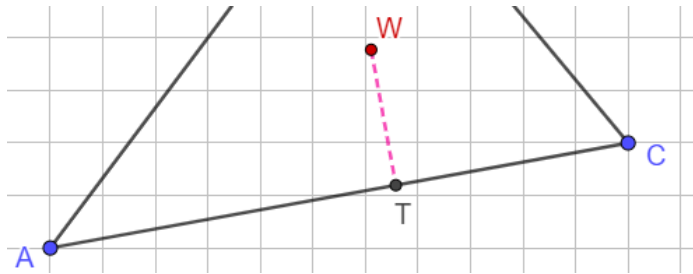
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**Activity 2B**

**To ensure that the warehouse is equidistant from three roads.**

5. Between the three stores  $A$ ,  $B$  and  $C$ , there are three distributor roads that form  $\triangle ABC$ . Our boss is searching for a location to build our warehouse  $W$  and then construct paths from  $W$  to each distributor road. There are two requirements regarding the selected location:
- $W$  must be equidistant from each of the three distributor roads  $AB$ ,  $BC$  and  $AC$ .
  - To minimise travelling time, the paths from  $W$  to each of the three distributor roads must be the shortest.

In the following example, describe the geometrical relationship between road  $AC$  and path  $WT$ .




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6. Which of the centres of  $\triangle ABC$  can meet our boss's requirement?

Use the following applet to verify your answer.

Link: <https://www.geogebra.org/m/bgpwnvje>

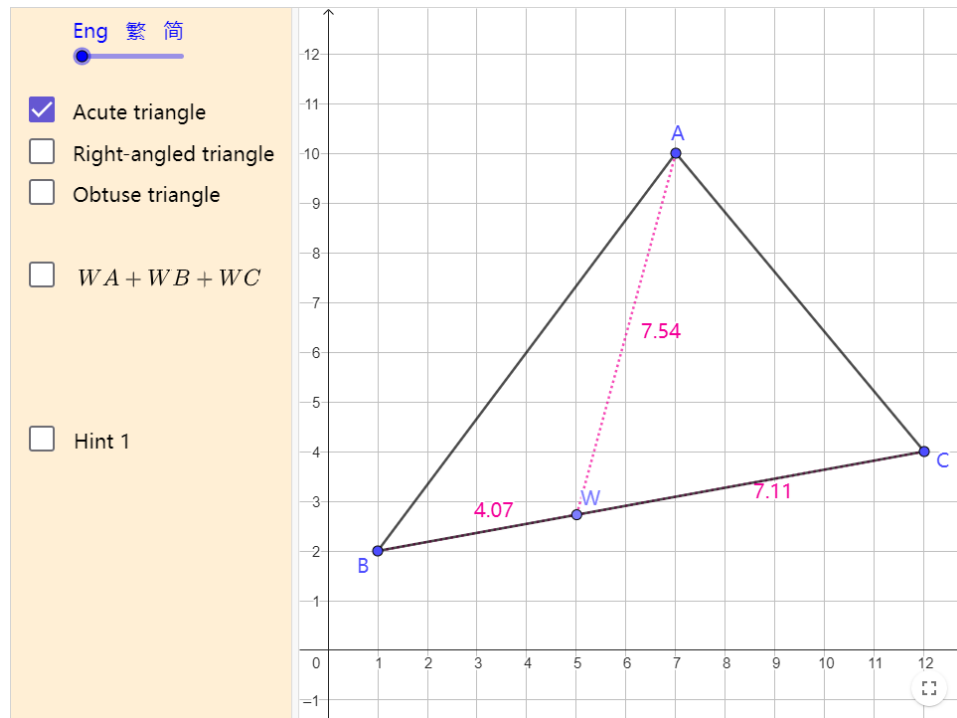
Your choice	Centres	Distance (correct to the nearest 0.01 km)		
		$WR$	$WS$	$WT$
<input checked="" type="checkbox"/>				
	Incentre			
	Circumcentre			
	Centroid			
	Orthocentre			

### Activity 2C

To minimise total distance between the warehouse and the three stores.

7. To better connect with the rapid transit system, an engineer suggests locating the warehouse  $W$  beside a distributor road. Nevertheless, the total distance between  $W$  and our three stores  $A$ ,  $B$  and  $C$  must be minimised.

Explore using the following applet: <https://www.geogebra.org/m/gxtnz4cu>



Sketch and describe how to find the location of the warehouse  $W$ .

*Hint: You will need to consider three different cases. It may be easier to start with the case in which  $\triangle ABC$  is a right-angled triangle.*

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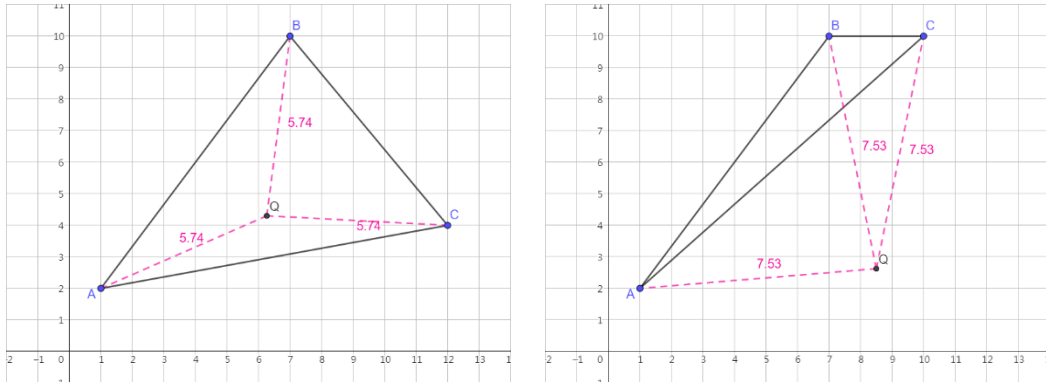
## Finding the optimal locations for company headquarters and warehouse

### Worksheet 3

#### Activity 3

To use information technology in modelling.

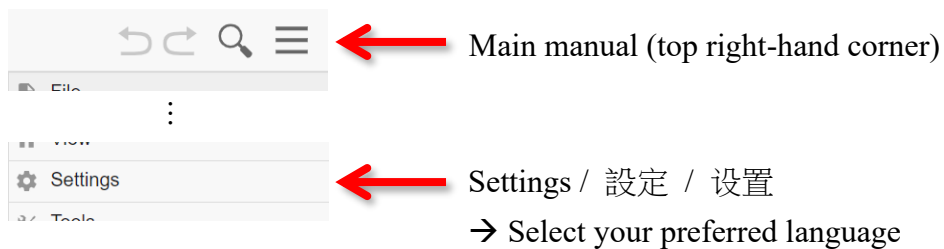
1. We can use GeoGebra to create a virtual model. It will be useful for presenting our modelling outcomes, such as the following.



If we build our headquarters  $Q$  at the circumcentre of  $\triangle ABC$ , then  $QA = QB = QC$ . But we found that when  $\triangle ABC$  is an obtuse triangle,  $Q$  lies outside the triangle.

Go to GeoGebra official website: <https://www.geogebra.org/classic>

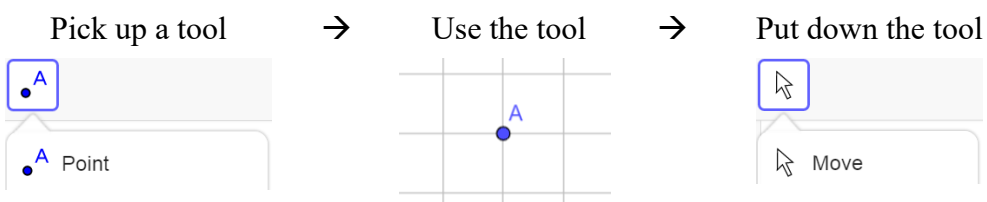
We can set the language of GeoGebra:



#### Tips

In GeoGebra, after using a tool, it is a good practice to select “Move” tool.

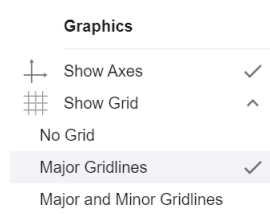
You can understand this practice as:



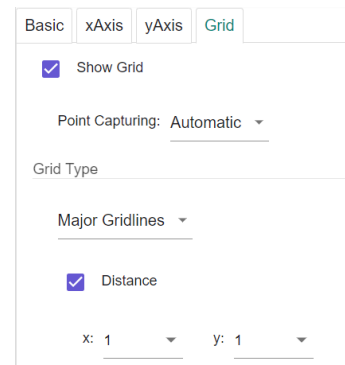
2. The following steps will guide us through locating our headquarters  $Q$  between stores  $A$  and  $B$  in Activity 1A. i.e., the mid-point of  $A$  and  $B$ .

Step	Description
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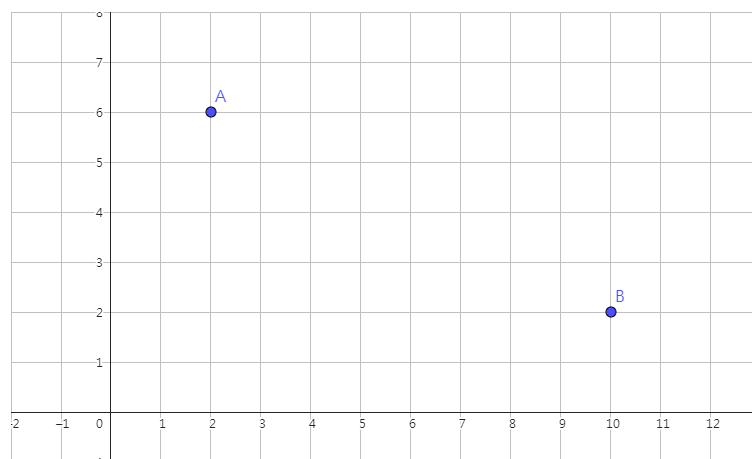
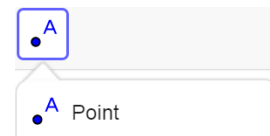
- i. Set the gridlines
- Right-click on the graphics view
  - “Show Grid” → Tick “Major Gridlines”



- ii. Set the distance of the grid
- Right-click on the graphics view
  - Click “Graphics”
  - Go to “Grid” tab  
→ “Grid Type” → Select “Major Gridlines”
  - Tick “Distance”
  - Set  $x = 1$  and  $y = 1$

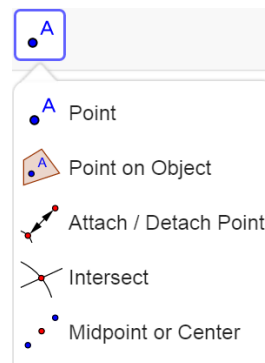


- iii. Locate the stores
- Use “Point” tool to draw Points  $A(2, 6)$  and  $B(10, 2)$  or anywhere that you want

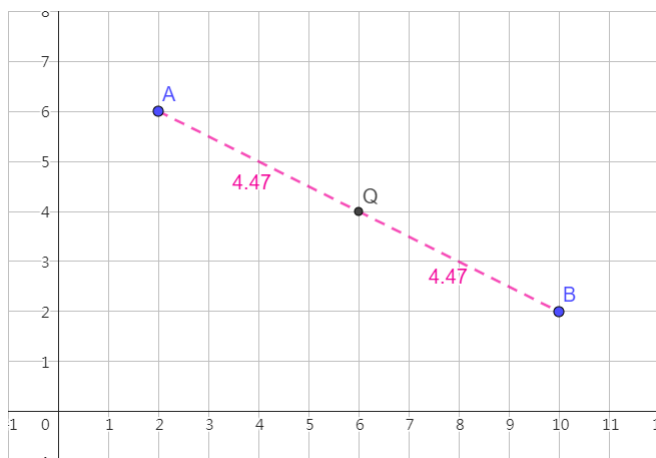


Step	Description
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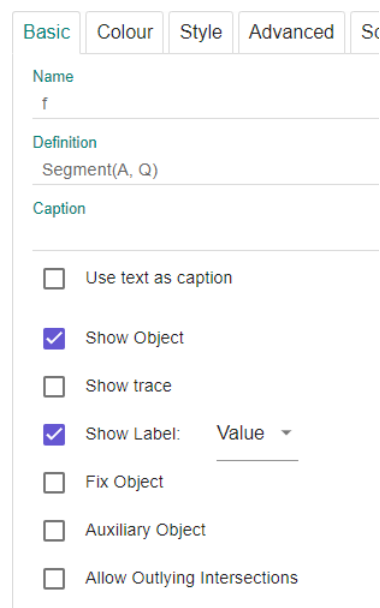
- iv. Locate the headquarters
- Use “Midpoint or Center” tool to locate our headquarters  
→ Select Point  $A$  → Select Point  $B$
  - Right-click the midpoint  
→ “Rename” → Input “ $Q$ ”



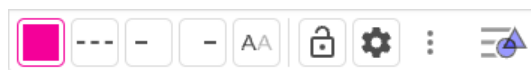
- v. Show the distance of  $QA$  and  $QB$
- Use “Segment” tool to draw Line segment  $QA$
  - Right-click  $QA$  → “Settings”  
→ “Basic” tab → “Show Label”  
→ Select “Value”
  - Similar steps for  $QB$



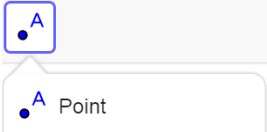
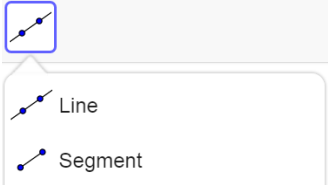
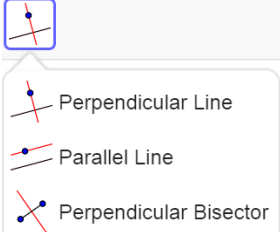
*Note:* You can move points  $A$  and  $B$  to observe the changes of point  $Q$ .

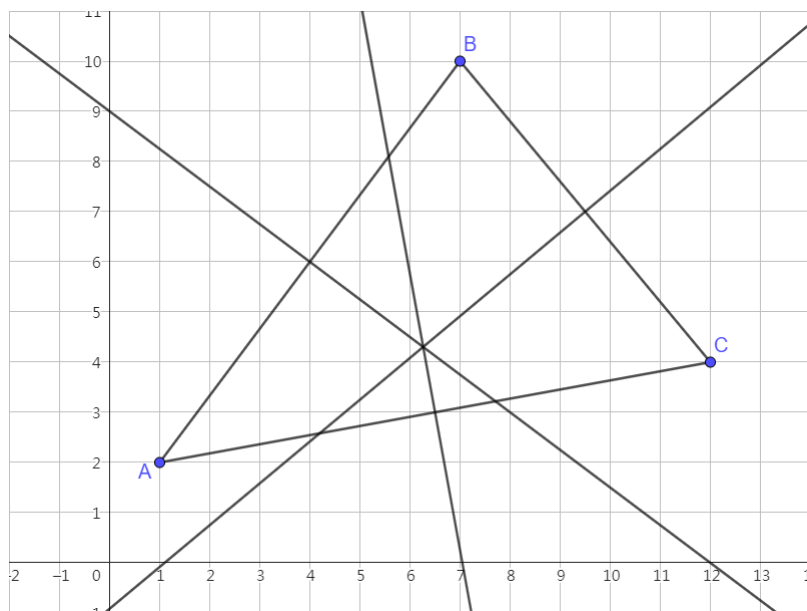


- vi. Set the colour and line style of  $QA$  and  $QB$
- Select  $QA$   
→ Click “Style Bar” (top-right corner)  
→ “Set colour” and “Set line style”
  - Similar steps for  $QB$



3. The following steps will guide us through locating our headquarters  $Q$  among stores  $A$ ,  $B$  and  $C$  in Activity 2A. i.e., the circumcentre of  $\triangle ABC$ .

Step	Description	
i.	Set the gridlines and the distance of the grid	(Same as Q2, steps i to ii)
ii.	Locate the stores <ul style="list-style-type: none"> <li>Use “Point” tool to draw Points <math>A(1, 2)</math>, <math>B(7, 10)</math> and <math>C(12, 4)</math> or anywhere that you want</li> </ul>	
iii.	Construct the distributor roads <ul style="list-style-type: none"> <li>Use “Segment” tool to draw Line segments <math>AB</math>, <math>BC</math> and <math>AC</math></li> </ul>	
iv.	Draw perpendicular bisectors of each side of the triangle <ul style="list-style-type: none"> <li>Use “Perpendicular Bisector” tool to draw the perpendicular bisector of <math>AB</math>  <math>\rightarrow</math> Select Point <math>A</math> <math>\rightarrow</math> Select Point <math>B</math></li> <li>Similar steps for <math>BC</math> and <math>AC</math></li> </ul>	





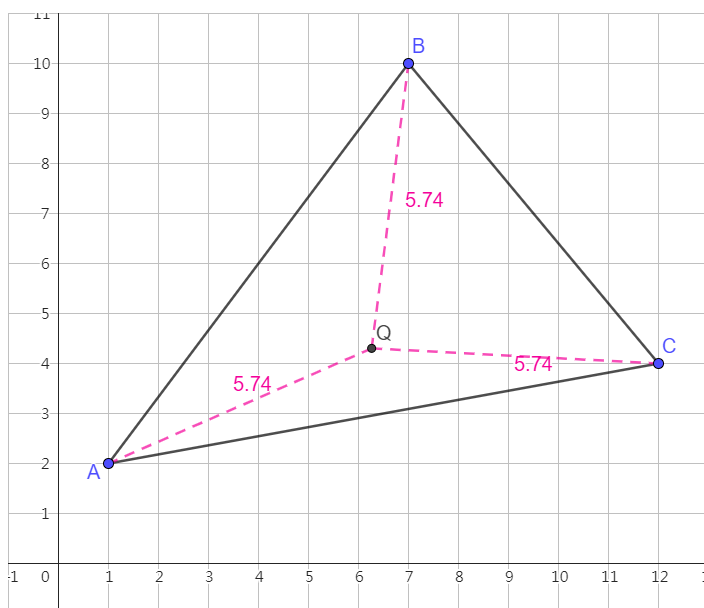
Step	Description
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- v. Locate the headquarters
- Use “Point” tool to draw the point of intersection of the three perpendicular bisectors
  - Right-click the point of intersection → “Rename” → Input “Q”

- vi. Hide the construction lines (i.e., the perpendicular bisectors)
- Right-click each perpendicular bisector → Untick “Show Object”

Line j: Perpendicular Bisector of AC	
Equation $y = m x + b$	
Parametric Form	
Equation $a x + b y + c = 0$	
<input type="checkbox"/>	Show Object
<input checked="" type="checkbox"/>	Show Label
<input type="checkbox"/>	Show trace

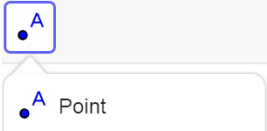
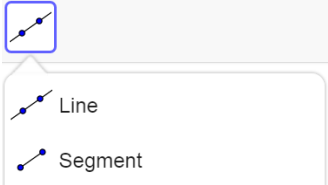
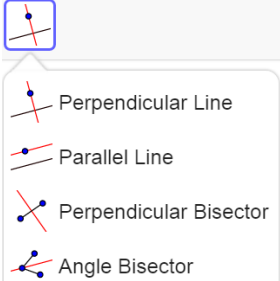
- vii. Show the distance of  $QA$ ,  $QB$  and  $QC$
- Use “Segment” tool to draw Line segment  $QA$
  - Right-click  $QA$  → “Settings” → “Basic” tab → “Show Label” → Select “Value”
  - Set the colour and line style of  $QA$
  - Similar steps for  $QB$  and  $QC$

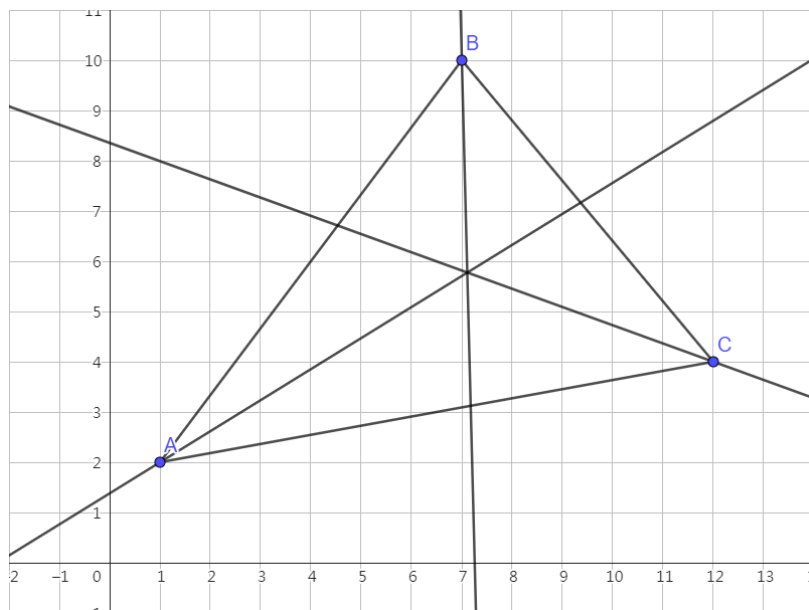


Basic	Colour	Style	Advanced	Sc
Name				
f				
Definition				
Segment(A, Q)				
Caption				
<input type="checkbox"/> Use text as caption				
<input checked="" type="checkbox"/> Show Object				
<input type="checkbox"/> Show trace				
<input checked="" type="checkbox"/> Show Label: Value ▾				
<input type="checkbox"/> Fix Object				
<input type="checkbox"/> Auxiliary Object				
<input type="checkbox"/> Allow Outlying Intersections				

Note: You can move points  $A$ ,  $B$  and  $C$  to observe the changes of point  $Q$ .

4. The following steps will guide us through locating our warehouse  $W$  among stores  $A$ ,  $B$  and  $C$  in Activity 2B. i.e., the incentre of  $\triangle ABC$ .

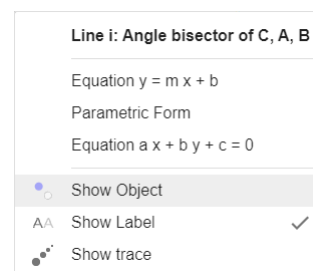
Step	Description	
i.	Set the gridlines and the distance of the grid	(Same as 2, steps i to ii)
ii.	Locate the stores <ul style="list-style-type: none"> <li>Use “Point” tool to draw Points <math>A(1, 2)</math>, <math>B(7, 10)</math> and <math>C(12, 4)</math> or anywhere that you want</li> </ul>	
iii.	Construct the distributor roads <ul style="list-style-type: none"> <li>Use “Segment” tool to draw Line segments <math>AB</math>, <math>BC</math> and <math>AC</math></li> </ul>	
iv.	Draw angle bisectors of each angle of the triangle <ul style="list-style-type: none"> <li>Use “Angle Bisector” tool to draw the angle bisector of <math>\angle BAC</math> <ul style="list-style-type: none"> <li>→ Select Point <math>B</math> → Select Point <math>A</math></li> <li>→ Select Point <math>C</math></li> </ul> </li> <li>Similar steps for <math>\angle ABC</math> and <math>\angle ACB</math></li> </ul>	



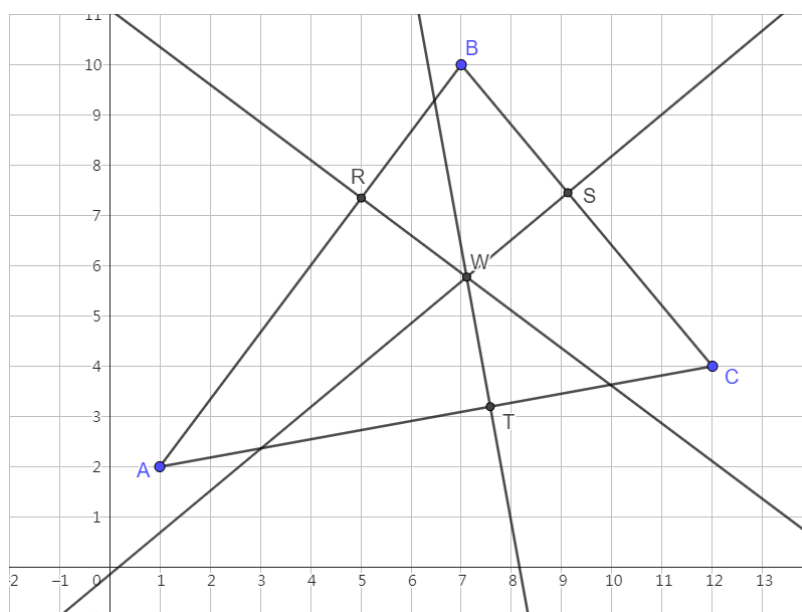
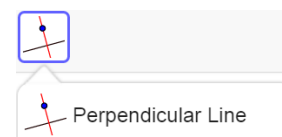
Step	Description
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- v. Locate the warehouse
- Use “Point” tool to draw the point of intersection of the three angle bisectors
  - Right-click the point  
→ “Rename” → Input “W”

- vi. Hide the construction lines (i.e., the angle bisectors)
- Right-click each angle bisector  
→ Untick “Show Object”

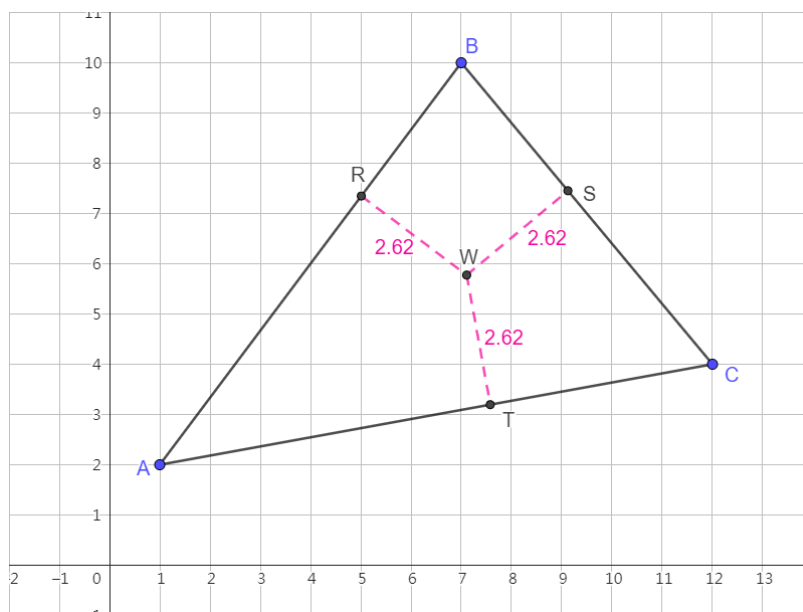


- vii. Draw the foot of a perpendicular from  $W$  on each side of the triangle
- Use “Perpendicular Line” tool  
→ Select Point  $W$  → Select  $AB$
  - Use “Point” tool to create the point of intersection
  - Similar steps for  $BC$  and  $AC$
  - Rename the points as in the figure



Step	Description
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- viii. Hide the construction lines (i.e., the perpendicular line)
- Right-click each perpendicular line  
→ Untick “Show Object”
- ix. Show the distance of  $WR$ ,  $WS$  and  $WT$
- Use “Segment” tool to draw  
Line segment  $WR$
  - Right-click  $WR$  → “Settings”  
→ “Basic” tab → “Show Label”  
→ Select “Value”
  - Set the colour and line style of  $WR$
  - Similar steps for  $WS$  and  $WT$



*Note:* You can move points  $A$ ,  $B$  and  $C$  to observe the changes of point  $W$ .



## Finding the Optimal Locations for Company Headquarters and Warehouse

Imagine you are the CEO of a company, tasked with finding the perfect locations for your new headquarters and warehouse. This task goes beyond simply picking spots on a map. It involves using mathematical modelling to make informed, strategic decisions that save time and transportation costs.

Consider the scenario in which your headquarters needs to be at the same shortest distance from two key stores. This is a manageable challenge, but it becomes more complicated when you have to consider three or more stores. For the warehouse, the challenge is to position it so that it is equidistant from major roads, optimising transportation routes and delivery times.

In this activity, you will learn to tackle these real-world problems. You will also identify and manage the constraints involved in such decisions. Let's embark on this quest to find the optimal locations for your company's headquarters and warehouse!

## Revision

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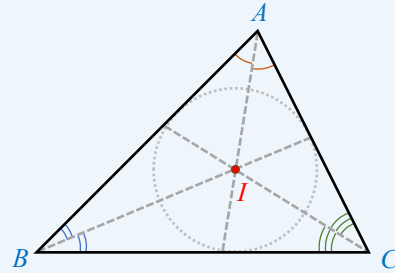
Perpendicular bisectors	Altitudes	Medians	Angle bisectors
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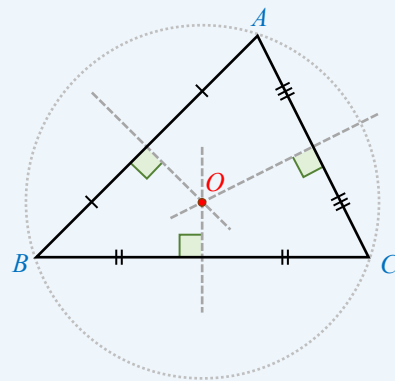
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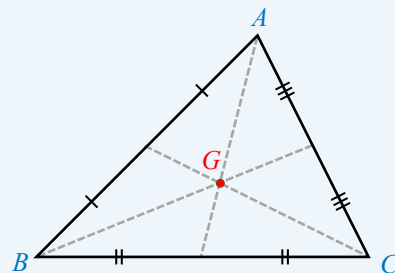
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in a triangle.

*Note:*  $O$  is the centre of the circle which passes through the three vertices of the triangle. The circle drawn is called the **circumscribed circle**.



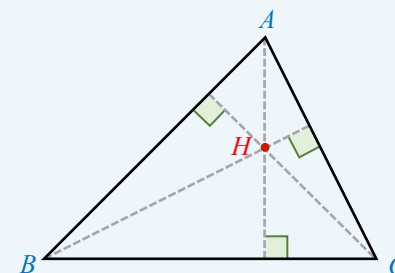
3. Centroid  $G$  is  
the point of intersection of

the three medians  
in a triangle.



4. Orthocentre  $H$  is  
the point of intersection of

the three altitudes  
in a triangle.



## Finding the optimal locations for company headquarters and warehouse

### Worksheet 1

#### Activity 1A

To ensure the same and shortest distance from the headquarters to two stores.

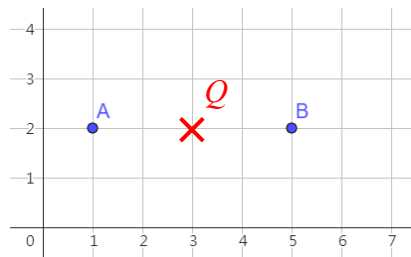
1. Headquarters  $Q$  is the main offices of our company, from where information is delivered to stores via radio technology. If there are two stores  $A$  and  $B$ , our boss wants to ensure that the distances between the headquarters and each of them are the same and the shortest.

Use mathematical terms to describe the requirement.

$Q$  is the mid-point of  $A$  and  $B$ .

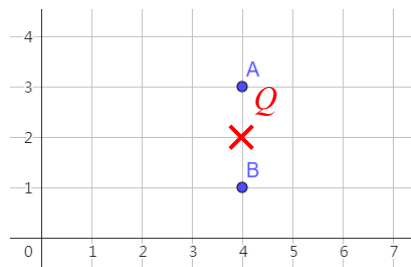
2. The following figures show some specific scenarios. Points  $A$  and  $B$  represent the locations of two stores, respectively. Mark the optimal locations of the headquarters  $Q$  on the figures and write down their coordinates.

(a)



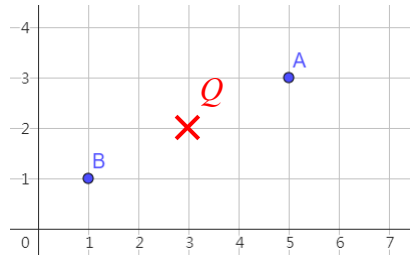
$$Q = ( 3 , 2 )$$

(b)



$$Q = ( 4 , 2 )$$

(c)



$$Q = ( 3 , 2 )$$

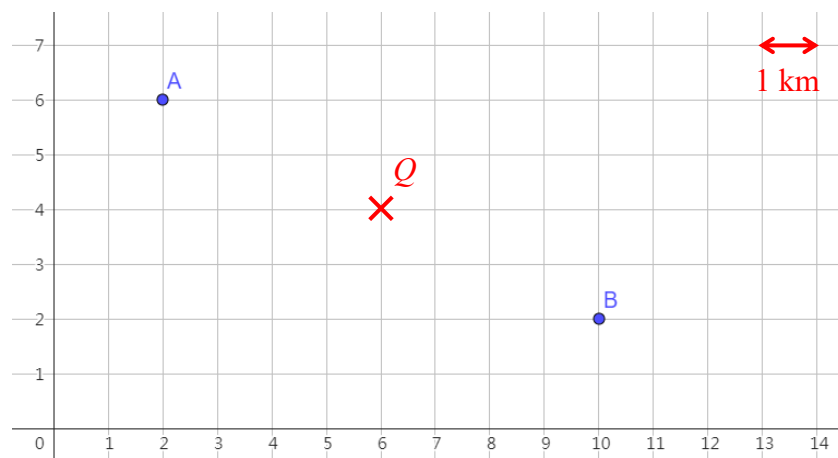
3. In reality, store locations vary across different regions.

Let the coordinates of two stores be  $(x_1, y_1)$  and  $(x_2, y_2)$ , respectively.

Write down a model of finding the optimal location for the headquarters  $Q$ .

$$Q = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

4. In the following figure, points  $A$  and  $B$  represent the locations of two stores, respectively.



- (a) Using the model in Question 3, find the coordinates of the headquarters  $Q$  and mark its location on the figure.
- (b) Calculate the distance from our headquarters to each of the stores correct to the nearest 0.01 km.

(a)  $A = (2, 6)$  and  $B = (10, 2)$

$$Q = \left( \frac{2+10}{2}, \frac{6+2}{2} \right)$$
$$= (6, 4)$$

(b)  $QA = \sqrt{(2-6)^2 + (6-4)^2}$

$$= \sqrt{20}$$
$$= 4.47 \text{ km}$$

$$QB = QA$$
$$= 4.47 \text{ km}$$



5. What assumptions are made in your model in Question 3?

- 2D-model: Assuming the ground is flat. In reality, the surface of the Earth is not flat. If the distances between the headquarters and the two stores are very long, we have to consider the curvature of the Earth's surface.
- Unobstructed transmission: There are no obstacles (e.g., hills or buildings) which block the transmission of the radio wave between the headquarters and the two stores.

6. Apart from the same and shortest distance from the headquarters to our stores, what factors should we consider when finding the optimal location for the headquarters?

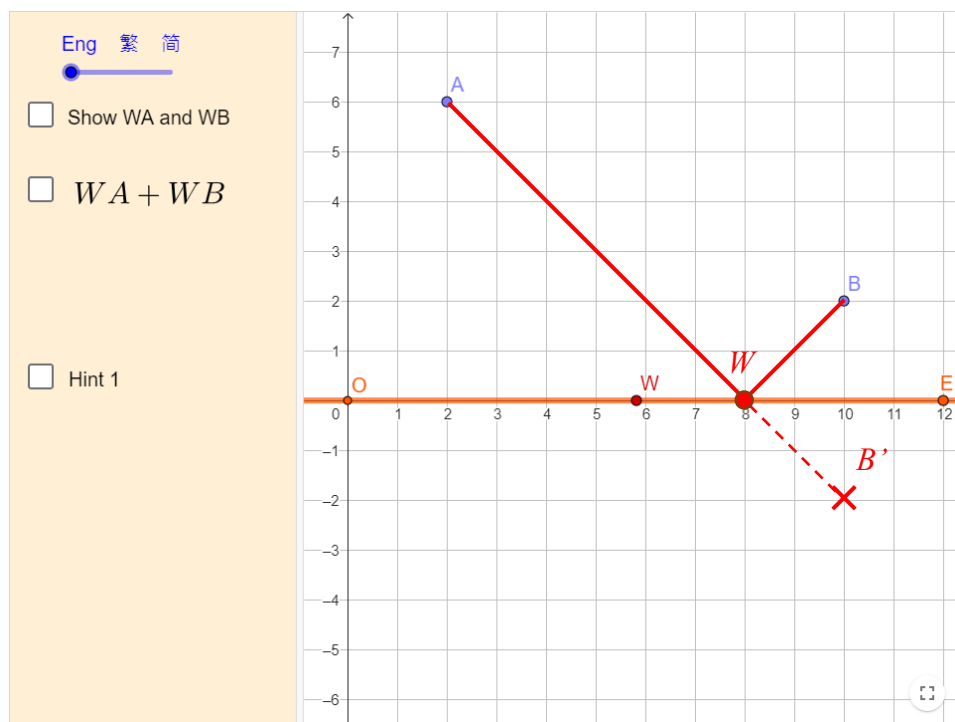
- Feasibility of construction: The construction of the headquarters at the selected location involves assessing whether the location meets the necessary building regulations and environmental considerations.
- Construction costs: The expenses associated with building the headquarters at the selected location include the costs of land acquisition, construction materials, and labour, etc.

## Activity 1B

To find the shortest distance between a warehouse and two stores.

7. Warehouse  $W$  is a large building for storing and distributing products to our stores. Our boss is searching for its best location. There are two requirements regarding the selected location:
  - i. To better connect with the rapid transit system,  $W$  must be located beside the distributor road  $OE$ .
  - ii. The total distance between  $W$  and our two stores  $A$  and  $B$  must be minimised. Nevertheless, the distances of paths  $WA$  and  $WB$  need not be the same.

Explore using the following applet: <https://www.geogebra.org/m/a6b9jb2g>



Based on the requirements,  $WA + WB$  should be the smallest (or shortest).

8. Describe how to find the location of the warehouse  $W$ .  
In the above figure, sketch your steps and mark the location of  $W$ .

First, Point  $B$  is reflected about  $OE$  and the image is  $B'$ .

Second, we draw a straight line  $AB'$ .

Then, the point of intersection of  $OE$  and  $AB'$  is the location of the warehouse  $W$ .

9. What are some possible constraints when searching for the location of the warehouse? How can we compromise?

- Possible constraints: The construction costs of building the warehouse beside a distributor road may be very high. In terms of city planning, the area close to a distributor road may be allocated for commercial or residential purposes, rather than industrial purposes.
- To remedy the constraints, we can first identify the optimal location of  $W$ . Then, we can search for other feasible locations as close to the optimal location as possible.

## Finding the optimal locations for company headquarters and warehouse

### Worksheet 2

#### Activity 2A

To ensure the same distance from the headquarters to three stores.

- If there are three different stores  $A$ ,  $B$  and  $C$ , our boss wants to ensure that the distances between the headquarters  $Q$  and each of them are the same.

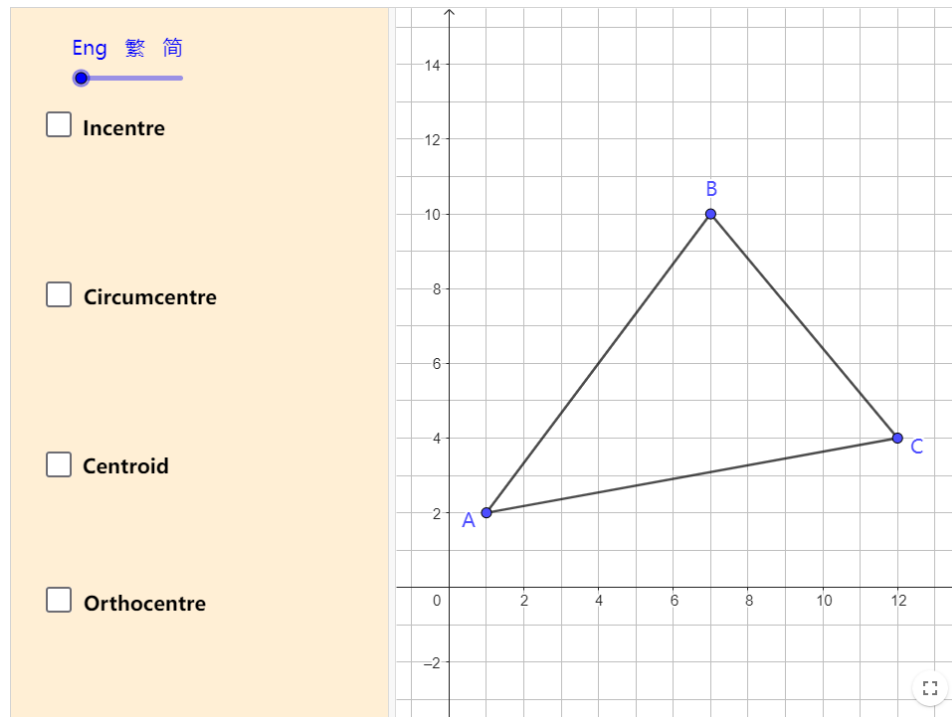
Write down the mathematical representation of this requirement.

$$QA = QB = QC \text{ (or } QA, QB \text{ and } QC \text{ are equal.)}$$

- Which of the centres of  $\triangle ABC$  can meet our boss's requirement?

Use the following applet to verify your answer.

Link: <https://www.geogebra.org/m/gm6ayhap>



Your choice	Centres	Distance (correct to the nearest 0.01 km)		
		$QA$	$QB$	$QC$
<input checked="" type="checkbox"/>				
	Incentre	7.18	4.23	5.20
✓	Circumcentre	5.74	5.74	5.74
	Centroid	6.57	4.68	5.50
	Orthocentre	8.43	2.65	5.66

3. What assumptions are made in finding the location?

- 2D-model: Assuming the ground is flat. In reality, the surface of the Earth is not flat. If the distances between the headquarters and the three stores are very long, we have to consider the curvature of the Earth's surface.
- Unobstructed transmission: There are no obstacles (e.g., hills or buildings) which block the transmission of the radio wave between the headquarters and the three stores.

4. Can the selected location minimise the total distance from our headquarters to the three stores?

You can use the following table to help explain your answer.

Centres	Distance (correct to the nearest 0.01 km)			
	$QA$	$QB$	$QC$	Total
Incentre	7.18	4.23	5.20	16.61
Circumcentre	5.74	5.74	5.74	17.22
Centroid	6.57	4.68	5.50	16.75
Orthocentre	8.43	2.65	5.66	16.74

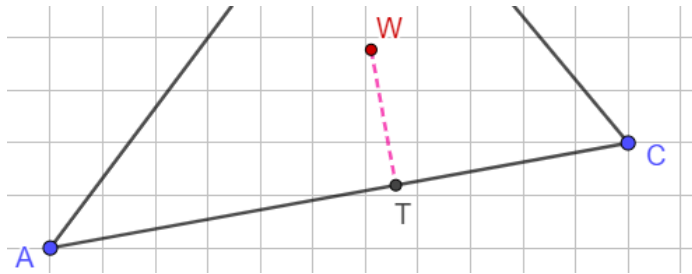
The selected location (the circumcentre of  $\triangle ABC$ ) cannot minimise the total distance from  $Q$  to  $A$ ,  $B$  and  $C$ . As shown in the above Table, the distance (circumcentre: 17.22 km) is greater than the three other locations, including the incentre (16.61 km), centroid (16.75 km) and orthocentre (16.74 km) of  $\triangle ABC$ .

## Activity 2B

To ensure that the warehouse is equidistant from three roads.

5. Between the three stores  $A$ ,  $B$  and  $C$ , there are three distributor roads that form  $\triangle ABC$ . Our boss is searching for a location to build our warehouse  $W$  and then construct paths from  $W$  to each distributor road. There are two requirements regarding the selected location:
- $W$  must be equidistant from each of the three distributor roads  $AB$ ,  $BC$  and  $AC$ .
  - To minimise travelling time, the paths from  $W$  to each of the three distributor roads must be the shortest.

In the following example, describe the geometrical relationship between road  $AC$  and path  $WT$ .



$WT \perp AC$  (or  $WT$  is perpendicular to  $AC$ .)

6. Which of the centres of  $\triangle ABC$  can meet our boss's requirement?

Use the following applet to verify your answer.

Link: <https://www.geogebra.org/m/bgpwnvje>

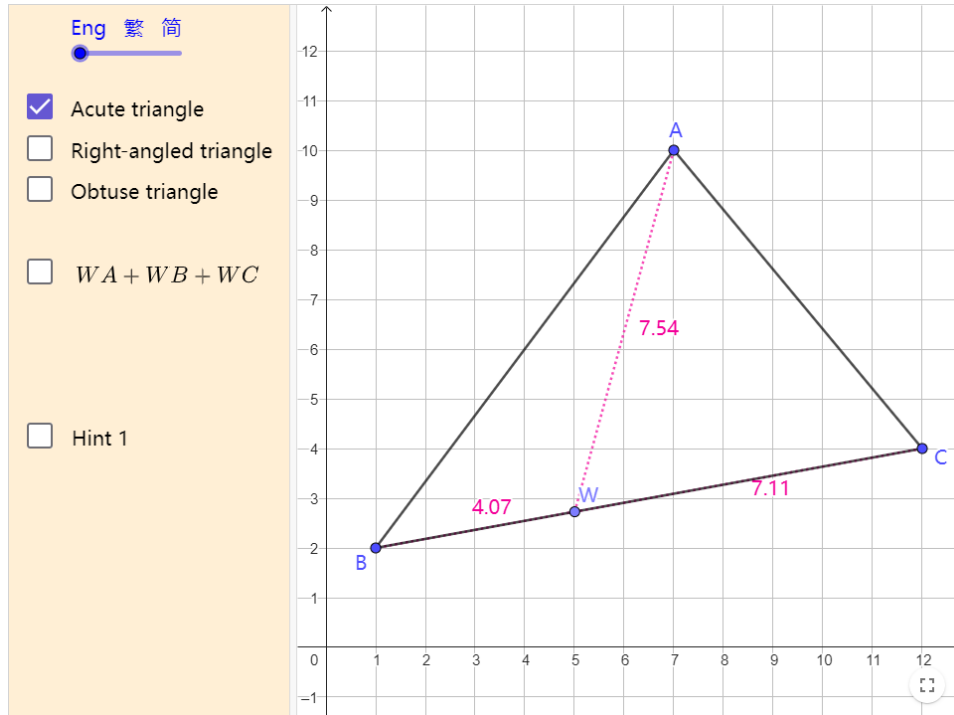
Your choice	Centres	Distance (correct to the nearest 0.01 km)		
		$WR$	$WS$	$WT$
<input checked="" type="checkbox"/>				
✓	Incentre	2.62	2.62	2.62
	Circumcentre	2.83	4.21	1.32
	Centroid	2.53	3.24	2.27
	Orthocentre	1.94	1.30	4.15

### Activity 2C

To minimise total distance between the warehouse and the three stores.

7. To better connect with the rapid transit system, an engineer suggests locating the warehouse  $W$  beside a distributor road. Nevertheless, the total distance between  $W$  and our three stores  $A$ ,  $B$  and  $C$  must be minimised.

Explore using the following applet: <https://www.geogebra.org/m/gxtnz4cu>

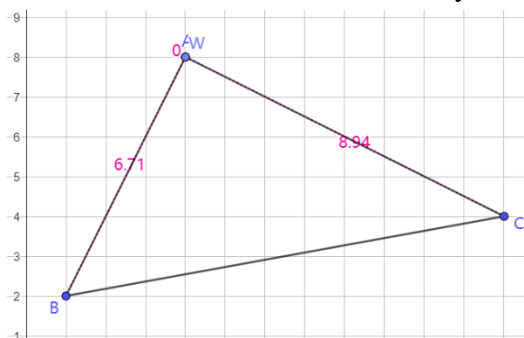


Sketch and describe how to find the location of the warehouse  $W$ .

*Hint: You will need to consider three different cases. It may be easier to start with the case in which  $\triangle ABC$  is a right-angled triangle.*

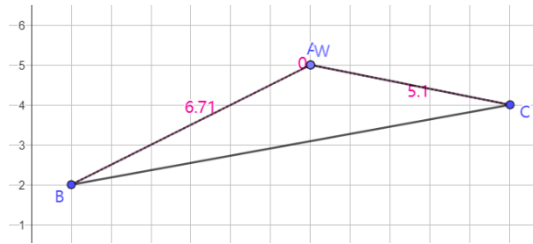
Case I: Right-angled triangle

$W$  is located at the vertex formed by the two shorter sides of the triangle.



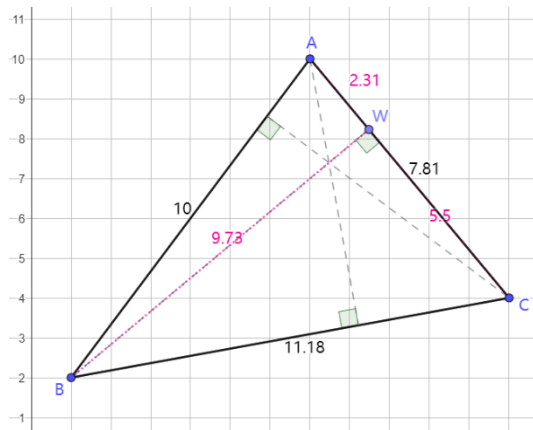
Case II: Obtuse triangle

$W$  is located at the vertex formed by the two shorter sides of the triangle.



Case III: Acute triangle

$W$  is located at the foot of a perpendicular on the shortest side (from its corresponding vertex) of the triangle.





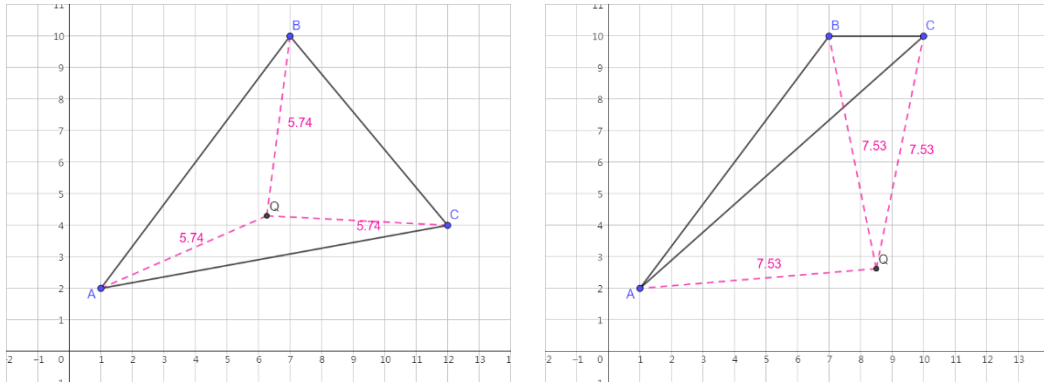
## Finding the optimal locations for company headquarters and warehouse

### Worksheet 3

#### Activity 3

To use information technology in modelling.

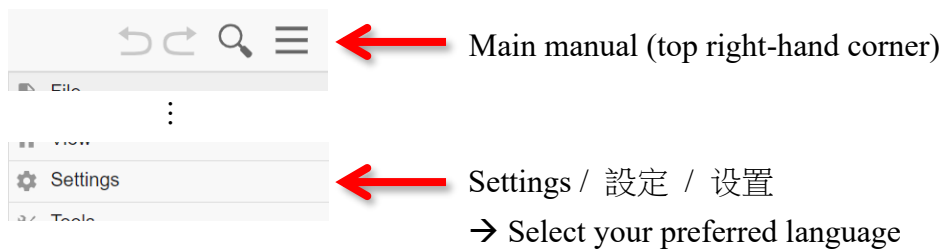
1. We can use GeoGebra to create a virtual model. It will be useful for presenting our modelling outcomes, such as the following.



If we build our headquarters  $Q$  at the circumcentre of  $\triangle ABC$ , then  $QA = QB = QC$ . But we found that when  $\triangle ABC$  is an obtuse triangle,  $Q$  lies outside the triangle.

Go to GeoGebra official website: <https://www.geogebra.org/classic>

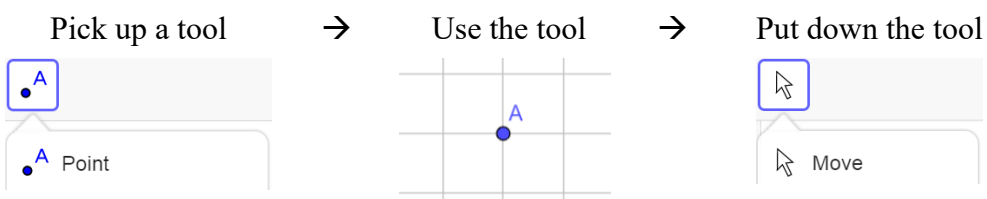
We can set the language of GeoGebra:



#### Tips

In GeoGebra, after using a tool, it is a good practice to select “Move” tool.

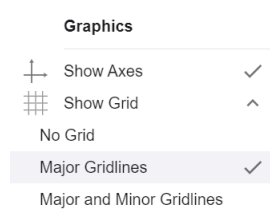
You can understand this practice as:



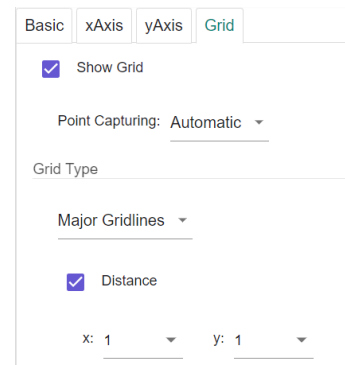
2. The following steps will guide us through locating our headquarters  $Q$  between stores  $A$  and  $B$  in Activity 1A. i.e., the mid-point of  $A$  and  $B$ .

Step	Description
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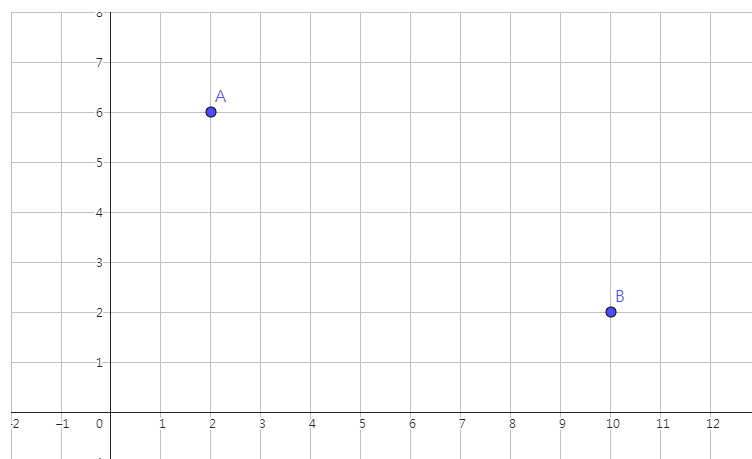
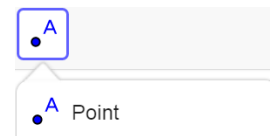
- i. Set the gridlines
- Right-click on the graphics view
  - “Show Grid” → Tick “Major Gridlines”



- ii. Set the distance of the grid
- Right-click on the graphics view
  - Click “Graphics”
  - Go to “Grid” tab
  - → “Grid Type” → Select “Major Gridlines”
  - Tick “Distance”
  - Set  $x = 1$  and  $y = 1$

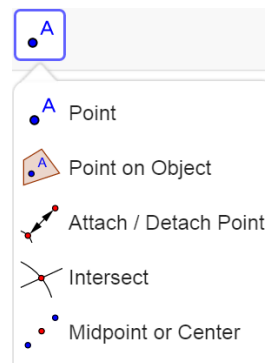


- iii. Locate the stores
- Use “Point” tool to draw Points  $A(2, 6)$  and  $B(10, 2)$  or anywhere that you want

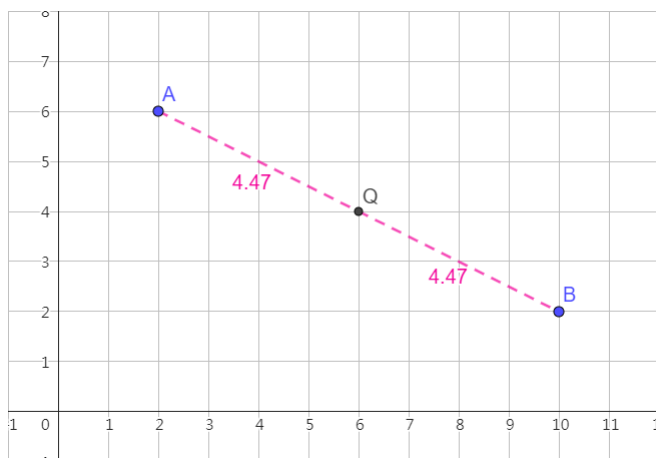


Step	Description
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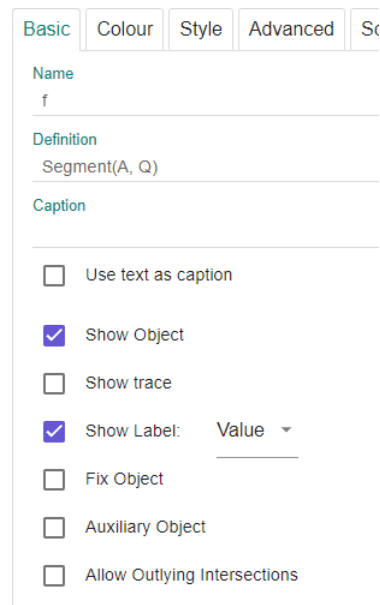
- iv. Locate the headquarters
- Use “Midpoint or Center” tool to locate our headquarters  
→ Select Point  $A$  → Select Point  $B$
  - Right-click the midpoint  
→ “Rename” → Input “ $Q$ ”



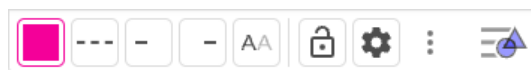
- v. Show the distance of  $QA$  and  $QB$
- Use “Segment” tool to draw Line segment  $QA$
  - Right-click  $QA$  → “Settings”  
→ “Basic” tab → “Show Label”  
→ Select “Value”
  - Similar steps for  $QB$



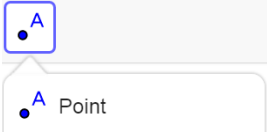
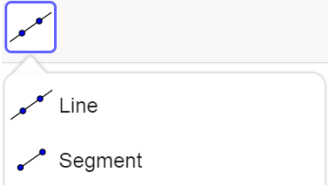
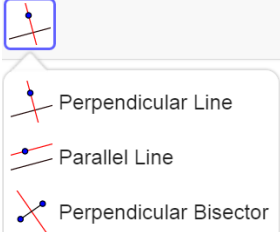
*Note:* You can move points  $A$  and  $B$  to observe the changes of point  $Q$ .

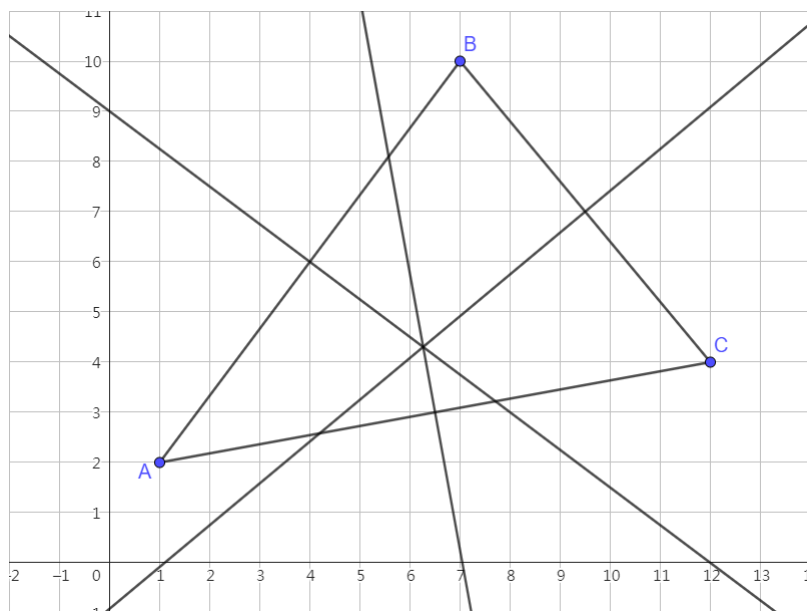


- vi. Set the colour and line style of  $QA$  and  $QB$
- Select  $QA$   
→ Click “Style Bar” (top-right corner)  
→ “Set colour” and “Set line style”
  - Similar steps for  $QB$



3. The following steps will guide us through locating our headquarters  $Q$  among stores  $A$ ,  $B$  and  $C$  in Activity 2A. i.e., the circumcentre of  $\triangle ABC$ .

Step	Description	
i.	Set the gridlines and the distance of the grid	(Same as Q2, steps i to ii)
ii.	Locate the stores <ul style="list-style-type: none"> <li>Use “Point” tool to draw Points <math>A(1, 2)</math>, <math>B(7, 10)</math> and <math>C(12, 4)</math> or anywhere that you want</li> </ul>	
iii.	Construct the distributor roads <ul style="list-style-type: none"> <li>Use “Segment” tool to draw Line segments <math>AB</math>, <math>BC</math> and <math>AC</math></li> </ul>	
iv.	Draw perpendicular bisectors of each side of the triangle <ul style="list-style-type: none"> <li>Use “Perpendicular Bisector” tool to draw the perpendicular bisector of <math>AB</math>                → Select Point <math>A</math> → Select Point <math>B</math></li> <li>Similar steps for <math>BC</math> and <math>AC</math></li> </ul>	

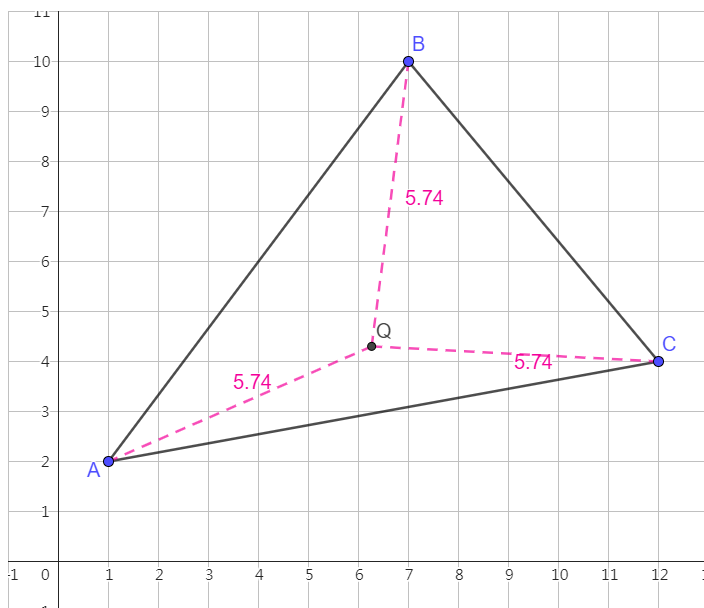


Step	Description
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- v. Locate the headquarters
- Use “Point” tool to draw the point of intersection of the three perpendicular bisectors
  - Right-click the point of intersection → “Rename” → Input “Q”
- vi. Hide the construction lines (i.e., the perpendicular bisectors)
- Right-click each perpendicular bisector → Untick “Show Object”

Line j: Perpendicular Bisector of AC	
Equation $y = m x + b$	
Parametric Form	
Equation $a x + b y + c = 0$	
<input type="checkbox"/>	Show Object
<input checked="" type="checkbox"/>	Show Label
<input type="checkbox"/>	Show trace

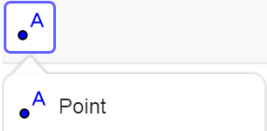
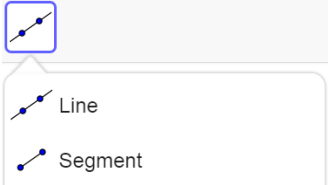
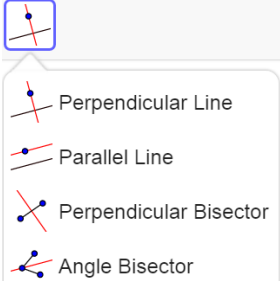
- vii. Show the distance of  $QA$ ,  $QB$  and  $QC$
- Use “Segment” tool to draw Line segment  $QA$
  - Right-click  $QA$  → “Settings” → “Basic” tab → “Show Label” → Select “Value”
  - Set the colour and line style of  $QA$
  - Similar steps for  $QB$  and  $QC$

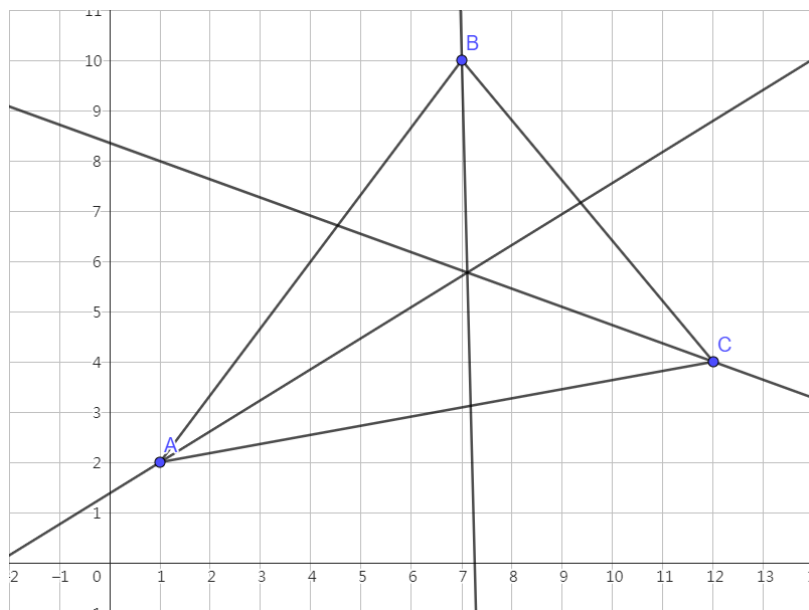


Basic	Colour	Style	Advanced	Sc
Name				
f				
Definition				
Segment(A, Q)				
Caption				
<input type="checkbox"/> Use text as caption				
<input checked="" type="checkbox"/> Show Object				
<input type="checkbox"/> Show trace				
<input checked="" type="checkbox"/> Show Label: Value				
<input type="checkbox"/> Fix Object				
<input type="checkbox"/> Auxiliary Object				
<input type="checkbox"/> Allow Outlying Intersections				

Note: You can move points  $A$ ,  $B$  and  $C$  to observe the changes of point  $Q$ .

4. The following steps will guide us through locating our warehouse  $W$  among stores  $A$ ,  $B$  and  $C$  in Activity 2B. i.e., the incentre of  $\triangle ABC$ .

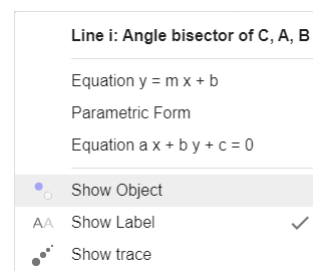
Step	Description	
i.	Set the gridlines and the distance of the grid	(Same as 2, steps i to ii)
ii.	Locate the stores <ul style="list-style-type: none"> <li>Use “Point” tool to draw Points <math>A(1, 2)</math>, <math>B(7, 10)</math> and <math>C(12, 4)</math> or anywhere that you want</li> </ul>	
iii.	Construct the distributor roads <ul style="list-style-type: none"> <li>Use “Segment” tool to draw Line segments <math>AB</math>, <math>BC</math> and <math>AC</math></li> </ul>	
iv.	Draw angle bisectors of each angle of the triangle <ul style="list-style-type: none"> <li>Use “Angle Bisector” tool to draw the angle bisector of <math>\angle BAC</math> <ul style="list-style-type: none"> <li>→ Select Point <math>B</math> → Select Point <math>A</math></li> <li>→ Select Point <math>C</math></li> </ul> </li> <li>Similar steps for <math>\angle ABC</math> and <math>\angle ACB</math></li> </ul>	



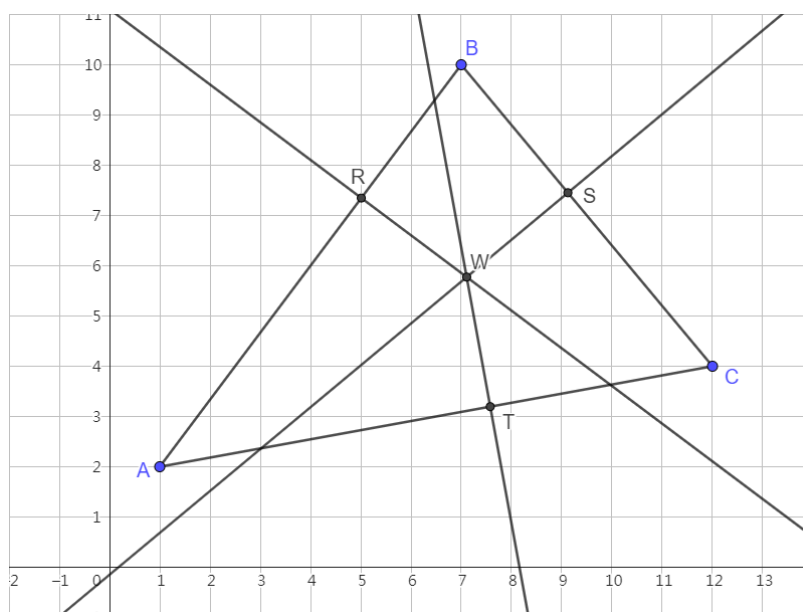
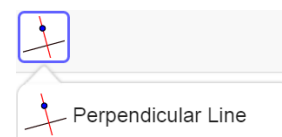
Step	Description
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- v. Locate the warehouse
- Use “Point” tool to draw the point of intersection of the three angle bisectors
  - Right-click the point  
→ “Rename” → Input “W”

- vi. Hide the construction lines (i.e., the angle bisectors)
- Right-click each angle bisector  
→ Untick “Show Object”

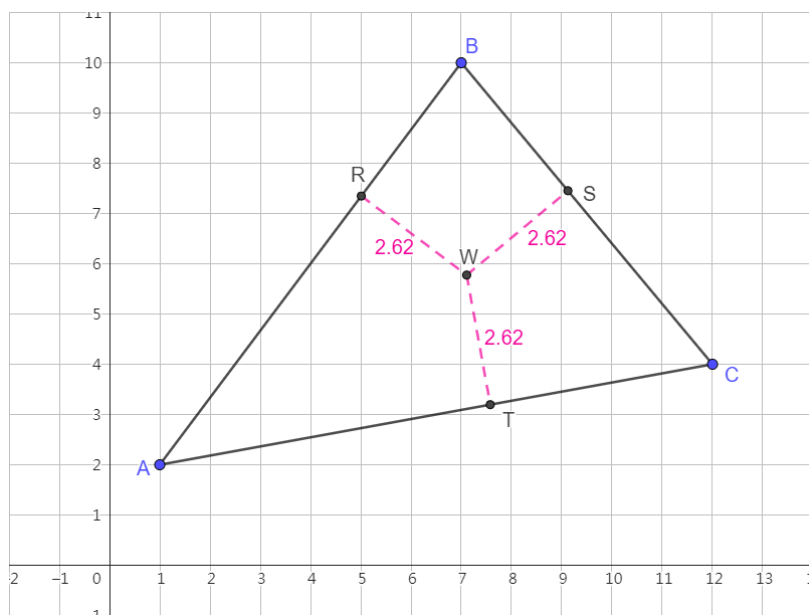


- vii. Draw the foot of a perpendicular from  $W$  on each side of the triangle
- Use “Perpendicular Line” tool  
→ Select Point  $W$  → Select  $AB$
  - Use “Point” tool to create the point of intersection
  - Similar steps for  $BC$  and  $AC$
  - Rename the points as in the figure



Step	Description
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- viii. Hide the construction lines (i.e., the perpendicular line)
- Right-click each perpendicular line  
→ Untick “Show Object”
- ix. Show the distance of  $WR$ ,  $WS$  and  $WT$
- Use “Segment” tool to draw  
Line segment  $WR$
  - Right-click  $WR$  → “Settings”  
→ “Basic” tab → “Show Label”  
→ Select “Value”
  - Set the colour and line style of  $WR$
  - Similar steps for  $WS$  and  $WT$



*Note:* You can move points  $A$ ,  $B$  and  $C$  to observe the changes of point  $W$ .