Learning To Learn

Key Learning Area

MATHEMATICS EDUCATION

Consultation Document

Hong Kong Special Administrative Region of
The People's Republic of China
Curriculum Development Council
November 2000

CONTENTS

1 INTRODUCTION

This document on the key learning area of Mathematics Education is written in support of the consultation document ***Learning to Learn*** prepared by the Curriculum Development Council (Nov 2000) and should be read together with it. The ***Learning to Learn*** document is the outcome of the Holistic Review of the School Curriculum conducted by CDC beginning in 1999, which is done in parallel with the Education Commission's Education System Review.

2 BACKGROUND

2.1 Students require knowledge and skills that will help them compete in the society of the 21st century, which is an information age. To reflect the rapid growth of knowledge in the information age, mathematics has unquestionably become a necessity for every individual to contribute towards the prosperity of the society. Mathematics pervades all aspects of life and it would be very difficult to live a normal life without making use of mathematics of some kind. Many of the developments and decisions made in industry and commerce, the provision of social and community services as well as government policy and planning etc., rely to some extent on the use of mathematics.

2.2 Mathematics is essential in the school curriculum of Hong Kong as it is:

- a powerful means of communication – It can be used to present information in many ways like figures, tables, charts, graphs and symbols, which can be further manipulated to deduce further information.
- an analyzing tool for studying other disciplines – It helps students enhance their understanding of the world.
- an intellectual endeavour and a mode of thinking – It is a creative activity in which students can be fully involved and display their imagination, initiative and flexibility of mind.
- a discipline, which can develop students' abilities to appreciate the beauty of nature, think logically and make sound judgement – These mathematical experiences acquired in schools enable students to become mathematically literate citizens and contribute towards the prosperity of the society.

Mathematics is valuable to help students develop the core competencies for lifelong learning. It is an integral part of the general education and hence a key learning area (KLA) of the school curriculum of Hong Kong.

3 RATIONALE FOR DEVELOPMENT

3.1 The CDC Committee on Mathematics Education holds the following views:

- High technology like computers and calculators has profoundly changed the world of mathematics education. Students should master information technology to become more adaptive to the dynamically changing environment. Mechanical drilling and impractical topics are no longer essential and relevant in mathematics learning.
- It is important for our students to gain experience and acquire foundation knowledge and skills, develop capabilities to learn how to learn, think logically and creatively, develop and use knowledge, analyze and solve problems, access and process information, make sound judgements and communicate with others effectively.
- Students should be able to build up confidence and positive attitudes towards mathematics learning, to value mathematics and to appreciate the beauty of mathematics.

3.2 According to two research studies[1], which were conducted in 1998 for supporting a holistic review[2] of the Hong Kong mathematics curriculum, although the current mathematics curriculum is well supported by various stakeholders, there are problems which should be resolved for improvement: The existing school mathematics curriculum is generally content-oriented[3], rather packed and difficult. The mathematics curriculum at S.1 repeats some of the material at P.5 and 6. Some of the content areas in Additional Mathematics overlap

---

[1] The two research studies were Comparative Studies of the Mathematics Curricula of Major Asian and Western Countries and An Analysis of the Views of Various Sectors on the Mathematics Curriculum. The former was conducted by The University of Hong Kong while the latter The Chinese University of Hong Kong. The two research studies were completed with reports compiled in mid-1999. Summaries of the findings are listed in Appendix 1 and the reports can be found from the web site www.cdccdi.hk.linkage.net/cdi/maths/ index.htm.

[2] The holistic review was carried out by an Ad hoc Committee, which was set up in July 1997 by the former CDC. The chairman was Dr Wong King-keung who was a member of the Hong Kong Airport Authority. The final report of the Ad hoc Committee had been issued to schools in June 2000 and can be found from the web site www.cdccdi.hk.linkage.net/cdi/ maths/index.htm.

[3] The term "content" used in this consultative document means "knowledge objectives" unless otherwise stated.

with those of Secondary Mathematics (1985), AS-level Mathematics & Statistics and A-level Pure Mathematics. Such redundancy has brought difficulties to teachers teaching these subjects and generated an unnecessary inclination among students to take Additional Mathematics even if they do not have the competence. Moreover, the similarity between A-level and AS-level Applied Mathematics, the lengthiness of A-level Pure Mathematics and the unclear target students of AS-level Mathematics & Statistics have drawn grievances from the school teachers.

3.3 Teaching at the senior primary, senior secondary and sixth form levels is examination-driven. Most teachers and students only focus on the teaching and learning of those materials which are included in the examination syllabuses. Mathematical knowledge which can arouse students' interest is seldom introduced if it is beyond the examination syllabuses. The scope of learning is therefore confined.

4 PHASES OF DEVELOPMENT

4.1 Short-term (2000 – 2005)

4.1.1 Both the primary and secondary mathematics curricula have been revised with the aim of shifting the emphasis of students from rote procedures and meaningless drilling to the development of their thinking abilities, catering for the different needs and abilities of students and strengthening their learning. In terms of content, the revised primary and secondary mathematics curricula have been trimmed down by about 15% and 11% respectively. If necessary, schools / teachers could further adapt the curricula to create more space for

- using IT in mathematics teaching / learning;
- conducting project learning;
- doing exploratory work;
- organizing consolidation / enrichment activities, etc.

4.1.2 Tryouts of some selected topics (see Appendix 2 for the list of tryout topics / teaching strategies) in some schools have been conducted since 1997. The results of the tryouts and the comments and suggestions from the schools concerned, teachers' opinions from related surveys and seminars, and the recommendations of the Ad hoc Committee were adequately considered before the finalization of the revised curricula. The revised secondary mathematics curriculum[4] will be implemented at S.1 in September 2001 and the primary one[5] at P.1 in September 2002.

4.1.3 The Education Department will organize relevant in-service training programs to familarize teachers with the focuses of the revised curricula. Reference materials including exemplars, useful web sites and CD Roms will also be produced.

---

[4] The curriculum document *Syllabuses for Secondary Schools: Mathematics (Secondary 1-5) (1999)* was issued to secondary schools in March 2000.

[5] The curriculum document *Syllabuses for Primary Schools: Mathematics (Primary 1-6) (2000) (Second Draft)* has been endorsed by CDCC on Mathematics Education and will be issued to primary schools in April / May 2001.

4.2 Medium-term (2005 – 2010)

To reduce the effect of overlapping between Additional Mathematics and the related mathematics subjects at the sixth form levels, the curriculum of Additional Mathematics will be revised in the near future.

4.3 Long-term (2010+)

In the long term, pending the review of the new senior secondary structure, the sixth form mathematics curriculum will be re-structured to cope with the revised primary and secondary mathematics curricula, to ensure better continuity among various mathematics subjects and to suit the diversified needs of students.

5 THE FRAMEWORK

5.1 Overall Aims

The aims of mathematics education are to develop:

- our youngsters' knowledge, skills and concepts of mathematics and to enhance their confidence and interest in mathematics, so that they can master mathematics effectively and are able to formulate and solve problems from a mathematical perspective; and
- their thinking abilities and positive attitudes towards learning mathematics and build related generic skills[6] throughout their life time.

5.2 Learning Targets

Both learning processes and content of mathematics are important in mathematics learning and hence reflected in the learning targets of the mathematics curriculum. Details of the learning targets in the four key stages can be found in Appendix 3.

5.3 Components of the Framework

Diagrammatic Representation of the Framework of the Mathematics Curriculum

In general, the framework of the mathematics curriculum can be represented diagrammatically as shown in Figure 1 on page 9.

[6] The 9 essential generic skills identified are collaboration skills, communication skills, creativity, critical thinking skills, information technology skills, numeracy skills, problem solving skills, self management skills and study skills (see Appendix 4 for details).

5.3.1 *Strands*[7]

It can be seen from Figure 1 that the essential learning experiences for achieving the aims of mathematics education are organized into 5 learning dimensions at the primary level and 3 at the secondary.

Primary	*Secondary*
• Number	• Number & Algebra
• Algebra	• Measures, Shape & Space
• Measures	• Data Handling
• Shape & Space	
• Data Handling	

The learning dimensions are merged from 5 dimensions to 3 because less emphasis will be put on the Number and Measures dimensions at the secondary level. Moreover, it is not easy to include certain learning areas in one single dimension in Key Stages 3 and 4. For example, trigonometry involves measures of angles and lengths as well as spatial concepts. Similarly, problem-solving strategies usually interweave at the higher level and it is not easy to solve problems by using only numerical or algebraic approach.

On the other hand, since students use mathematics to a different extent as they move along the grade levels, the use of dimensions will not be extended to the sixth form mathematics curriculum.

[7] The term "learning dimension" has been used in the revised primary and secondary school mathematics syllabuses.

Diagrammatic Representation of the Framework of the Mathematics Curriculum

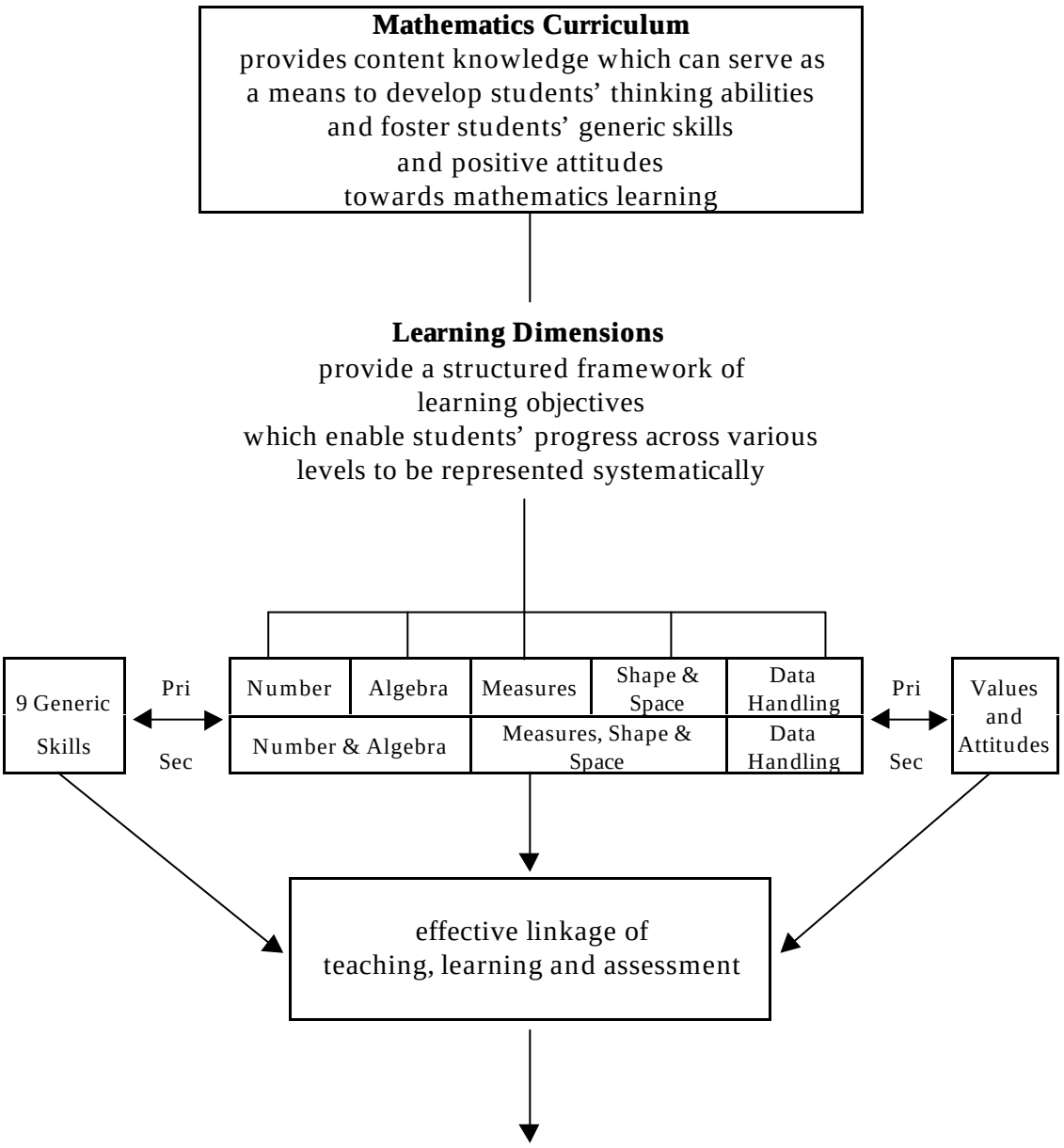


Figure 1

5.3.2 *Generic Skills, Values & Attitudes*

Daily life applications and high-order thinking skills (HOTs) are emphasized in the recently revised primary and secondary mathematics curricula. At the same time, as we live in a knowledge-based society in which many social policies and practices are shaped by statistically

manipulated data, the mastering of generic skills and the fostering of positive values and attitudes should also be stressed. They are expected to develop/be fostered through the learning of mathematical knowledge in the content areas and it is desirable for teachers to help students cultivate these learning elements through planned learning activities. Figure 2 illustrates how these learning elements intertwine to form a reference grid.

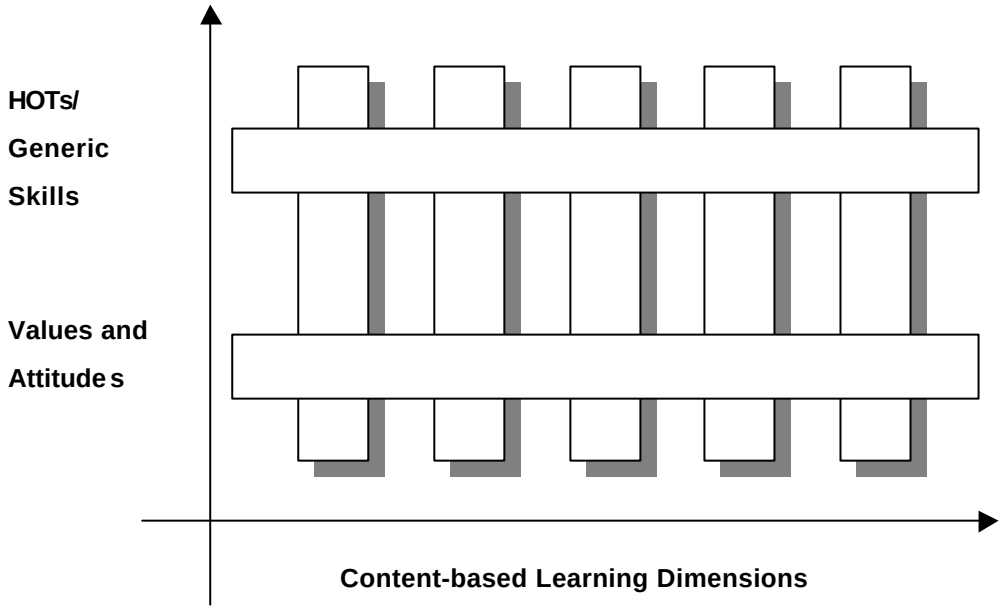


Figure 2

The list of generic skills, which are regarded as paramount for lifelong learning in a world where knowledge is ever changing, is attached in Appendix 4. Examples showing how the skills can be experienced through the course of mathematics learning are provided. Similarly, related values and attitudes which are expected to be developed throughout the mathematics course at various stages of the Mathematics Education are shown in Appendix 5. Appendix 6 gives six detailed exemplars which illustrate how the learning targets and generic skills can be linked. Exemplars 1 to 3 are at the primary level and exemplars 4 to 6 at the secondary level. The linkage between the exemplars and the main learning targets and generic skills is summarized in the following table.

Exemplar	*Main learning targets linked*	*Main generic skills linked*
1	"To understand … whole numbers" of the Number Dimension in Key Stage 1	• critical thinking • communication • problem solving
2	"To group and make … 3-dimensional shapes" of the Shape & Space Dimension in Key Stage 2	• communication • critical thinking • problem solving
3	"To formulate and solve problems arising from collected data and constructed graphs" of the Data Handling Dimension in Key Stage 2	• collaboration • critical thinking • problem solving • creativity • numeracy
4	"To interpret simple algebraic relations from numerical, symbolic and graphical perspectives" of the Number and Algebra Dimension in Key Stage 3	• information technology • numeracy • critical thinking • problem solving
5	"To investigate and describe relationships between quantities using algebraic symbols and relations" of the Number and Algebra Dimension in Key Stage 4	• information technology • numeracy • critical thinking
6	"To select and use the measures of central tendency and dispersion to compare data sets" of the Data Handling Dimension in Key Stage 4	• communication • numeracy • critical thinking

Schools could develop their own school-based mathematics curriculum along the same line as the framework with reference to the detailed set of learning objectives provided in the two recently revised Syllabuses.

5.4 Modes of Curriculum Planning

5.4.1 The principle of proposing different modes of curriculum planning is to build on strength of schools and teachers, and provide them with curriculum flexibility and diversity to meet different purposes of teaching/learning. At the primary and secondary levels, the curricula are planned by objectives and a "dimension approach" is adopted. Although the dimensions are content-based, the high-order thinking skills, generic skills, values and attitudes are expected to be incorporated into the content (see Figure 2) during teaching.

5.4.2 Enrichment activities/topics are provided in the revised primary mathematics curriculum, while the foundation part, non-foundation part and enrichment activities/topics are provided in the revised secondary mathematics curriculum to suit the different abilities of students.

5.4.3 Spare periods are also reserved at each key stage to provide teachers with curriculum space to rearrange or to adapt the content and depth of the teaching materials, using IT in mathematics teaching/learning and organize exploratory activities. Teachers have the flexibility to design their school-based mathematics curriculum to suit the needs of their students. However, it is not desirable to utilize the spare periods to do unnecessary drilling.

5.5 Teaching, Learning and Assessment

5.5.1 To take better account of the changing needs of the community, the main focus of the revised primary and secondary mathematics curricula is not on what mathematical topics should be learnt but on how mathematics is learnt. Therefore, the acquisition of high-order thinking skills and generic skills and the fostering of positive attitude towards mathematics learning are strongly advocated and should be allied with the learning of mathematical content. The teachers' role should be to facilitate students to learn how to learn through using the said skills in mathematics.

5.5.2 Teaching strategies should be progressively changed through different levels of schooling, say from concrete to abstract, so as to cope with students' development. A thematic approach is encouraged at junior primary levels. Similarly, investigational work is encouraged at all the primary levels and should be continued in secondary schools. The teaching of abstract mathematical ideas should be supported by students' concrete experiences at earlier stages as far as possible. This is because students need time to play around with concrete objects before proceeding to more abstract notions at the senior level. On the whole, exposure to concrete objects and gaining personal experiences are important and should therefore be planned and built into the teaching programme as much as possible to support discussion in the mathematics curriculum.

5.5.3 Diversified teaching/learning activities including projects are encouraged. The application of mathematical concepts receive greater attention as it provides students with motivation for learning mathematics and experiences in using mathematical models. Therefore, daily-life situations are suggested as the means to help students realize the need for mathematics and its applications. In the revised secondary mathematics curriculum, further applications of mathematical knowledge in more complex real-life situations, which require students to integrate their knowledge and skills from various disciplines to solve problems, are provided through the new module "Further Applications".

5.5.4 Since each student has his/her own strengths and weaknesses, the mathematics curriculum can be duly adjusted across different levels of learning to cater for the different abilities of students. The structures of the recently revised primary and secondary mathematics curriculum give teachers adequate flexibility to cater for the effects of learner differences. As well as the adaptation of the curriculum, information technology, diversified teaching approaches, remedial teaching, etc. can also be adopted. It is up to teachers' discretion to re-visit some topics when needed. However, it should be noted that though the spiral approach has its strong points,

teaching too many topics in a single year and making learning fragmented should be avoided. In addition, measures like organizing bridging programmes can be taken to ensure that students of different abilities can follow. In the process of developing the school-based curriculum, a flowchart indicating the inter-relation of topics at different levels is highly desirable to ensure continuity. The flowcharts attached in *Syllabuses for Primary Schools: Mathematics (Primary 1-6) (2000)* and *Syllabuses for Secondary Schools: Mathematics (Secondary 1-5) (1999)* can be referred to.

5.5.5 Apart from the formal mathematics curriculum, mathematics-related activities also play an important role in mathematics learning. It is generally agreed that well chosen and organized mathematics-related activities help to promote students' interest in learning the subject. Examples include mathematical games / puzzles, mathematics competitions / quizzes, mathematics workshops, projects, talks, plays, film shows, mathematics bulletins, newspaper cutting and board displays, etc.

5.5.6 Assessment is an integral part of the teaching-learning cycle. Valid and reliable assessment should reflect the objectives and goals of the curriculum. It should be used as a tool both to collect data and to influence instruction. Since student performance cannot be described by a single set of scores or by a single type of assessment activity because of its complexity, evidence of learning should be collected through various modes of assessment activities, to reflect students' achievement in mathematics. Formative assessments, which have come to be considered increasingly important in the teaching and learning processes, are also recommended, to provide a complete picture of student performance and help improve the learning of students. Students should be given a chance to demonstrate what they learn and how they apply their knowledge and skills of mathematics. Project work, class discussions, oral presentations and observations of students' performance during lessons are useful assessment activities and should be integrated with other classroom activities. A description of minimal competence, which is the pre-requisite to the learning of

mathematics at the next advanced stage, is helpful to teachers for reporting student performance in terms of the basic knowledge, concepts and skills acquired.

5.5.7 Communication is also one of the skills which students need to learn in mathematics. It is essential to assess students' ability in communicating findings, presenting an argument and explaining an intuitive approach to a problem either in an oral or a written form. From the written and oral presentation of students, teachers are able to identify their strengths and weaknesses. Over-drilling for examinations, on the other hand, will hamper effective learning and should thus be discouraged.

5.5.8 Schools need to formulate their assessment policy according to their culture, teachers' experiences, students' needs and interests. The design of learning objectives, learning activities and assessment tasks should be aligned to ensure that what is intended will be properly taught and successfully learned.

5.6 School-based Curriculum Development

Although both the revised primary and secondary mathematics curricula are recommended by CDC, schools are encouraged to carry out curriculum adaptation and integration, if deemed necessary, to meet the different needs, abilities and interests of students. The flexible elements introduced in the said curricula (i.e. foundation and non-foundation parts, and enrichment topics / activities) aim to help schools to design their school-based mathematics curriculum. For details, the two documents *Syllabuses for Primary Schools: Mathematics (Primary 1-6) (2000) (Second Draft)* and *Syllabuses for Secondary Schools: Mathematics (Secondary 1-5) (1999)* can be referred to.

5.7 Life-wide Learning

In addition to formal mathematics education in schools, there are also opportunities for students to acquire learning experiences in mathematics outside schools. Typical examples include seminars, activities and learning packages jointly

organized / developed by tertiary institutions / professional bodies / the government. Some popular activities are:

Activities	Specific Purposes	Organizing Bodies
Mathematics Competition for Hong Kong Primary Schools – a mathematics competition for primary students	The Competition aims to promote students' interest in studying mathematics and hence improve mathematics learning in primary schools.	Organized by the Professional Teachers' Union
Mathematics Trails – a mathematics competition for secondary students	Mathematics Trail aims to enhance students' mathematical problem solving skills through applications in real life situations and physical environment.	Organized by the HK Association of Science and Mathematics Education.
HK Mathematics Olympiad (HKMO) – a mathematics competition for secondary students	The Olympiad aims to promote and sustain students' interest in the study of mathematics.	Jointly organized by HKIEd and the Mathematics Section of ED.
Statistical Project Competition for Secondary School Students – a mathematics competition for secondary students	The Competition aims to promote students' interest in studying statistics and to encourage them to understand the community in a scientific manner through the use of statistics.	Jointly organized by the HK Statistical Society and the Statistics Department.

Po Leung Kuk Primary Mathematics World Contest – an international event for primary students	The Contest aims to discover mathematically gifted primary students. It also aims to create an opportunity for the exchange of learning experiences among gifted students.	Organized by the Po Leung Kuk. Training is provided to potential participants by Po Leung Kuk.
International Mathematical Olympiad – an international event for secondary students	The Olympiad aims to discover mathematically gifted young people who have not formally enrolled at any university. It also aims to foster friendly relations between gifted students.	Organized by the International Mathematical Olympiad (HK) Committee which is affiliated to the HK Mathematics Society. Training is provided to potential participants by the Committee.

More details can be found from the booklet "**全方位學習活動簡介**" (pp.8-9) published by "**教育統籌委員會**". The booklet can also be downloaded from the Internet at the web site http://www.life-wide-learning.org.

5.8 Connections with Other Key Learning Areas

5.8.1 Mathematics is the foundation and supporting knowledge to many other disciplines. It is linked to the other 7 KLAs by providing a basis for making investigations as well as a tool for analyzing data, representing findings and models with symbols, graphs and charts, and theorizing knowledge. On the other hand, other KLAs enrich students with examples of the applications of mathematics in real-life situations.

5.8.2 Students learn through direct experience. Therefore, to relate students' daily life experiences with mathematics, teachers have to organize real-life experiences in

mathematical ways, skills and instruments. For some selected topics (like percentage and statistics), integration with other KLAs (like Science or Personal, Social and Humanities) is one of the ways of organizing students' learning experiences mathematically. Integrated learning removes the boundaries of subjects and reflects the interdependent nature of reality and the complexities of life. It provides students with a holistic context for learning and enables students to make connection with the real world in solving problems. An example showing how this is done can be found in exemplar 3 in Appendix 6. Some of the links between the Mathematics KLA and the other KLAs are exemplified in the following paragraphs.

5.8.3 In the Chinese Language and English Language KLAs, mathematical concepts are essential to understand essays with mathematical and statistical ideas. For the Arts Education KLA, lines and shapes are important elements to create pictures and models, and patterns and symmetry are often explored in creative dance. In the Physical Education KLA, mathematics can help to analyze sport data and design an appropriate strategy for striving for sporting excellence.

5.8.4 In the Personal, Social and Humanities KLA, a variety of mathematical tools and procedures are used in making rational and responsible social decisions, such as identifying patterns and trends in statistical data and assessing validity in personal and social issues. Mathematical models are also used in theorizing knowledge in Social Sciences, in particular, Economics.

5.8.5 In the Science KLA, laws and formulae are represented in mathematical language, mathematical methods are employed to solve problems and generalize experimental findings and mathematical models are used to represent physical phenomena. In the Technology Education KLA, mathematical models are used in computer simulations to explore the feasibility of applying design ideas to investment decisions, and tables and charts are usually important tools to represent technical information.

6 CONCLUSION

6.1 To meet the challenges of an ever advancing knowledge-based society and a dynamically changing environment, mathematics has unquestionably become one of the necessities for enabling every individual to contribute towards the prosperity of society. The curriculum framework of the Mathematics Education KLA not only aims to provide students with mathematical knowledge, but also aims to equip them with a repertoire of skills, to help them develop thinking abilities and to foster positive attitudes, so that they can develop capabilities to learn how to learn and the confidence to face the knowledge-based society.

6.2 The revised secondary and primary mathematics curricula have been developed along these lines and will be implemented in September 2001 and September 2002 respectively. The sixth form mathematics curriculum will be revised once the review of the new senior secondary structure is clear. Additional Mathematics will be revised in the near future to enhance its continuity with the mathematics subjects concerned.

6.3 In this framework, diversified teaching/learning activities and adaptation of the curriculum to suit the different needs, abilities and interests of students are recommended. Multifarious assessment instruments can also be used to get complete profiles of students' performance. These profiles should be used to improve teaching/learning.

6.4 Apart from formal mathematics education in schools, mathematics-related activities organized by schools and local/international bodies also provide opportunities for students to acquire learning experiences in mathematics. Students should be encouraged to participate actively in these activities.

You are welcome to send your views to the Curriculum Development Council Secretariat by post, by fax or by e-mail on or before 15 February 2001.

Address: Curriculum Development Council Secretariat
Room 1329, Wu Chung House
213 Queen's Road East
Wan Chai
Hong Kong
Fax Number: 2573 5299 / 2575 4318
E-mail Address: cdchk@ed.gov.hk

Appendix 1

Findings of the Research Studies Conducted in 1998

Appendix 1

Findings of the Research Studies Conducted in 1998

The summaries[8] of the two research studies are as follows:

Research Study 1 – Comparative Studies of the Mathematics Curricula of Major Asian and Western Countries

The study consists of three components: a literature review, an analysis of curriculum documents, and a summary of the HK results in the TIMSS. The main findings are:

(a) The revised Secondary Mathematics Syllabus (1999) in HK is generally in line with worldwide trends.
(b) The HK mathematics curriculum attempts to strike a balance between process abilities (which are very much emphasized in the West) and basic skills and content (which are stressed in Asian countries).
(c) In HK, the introduction of topics into the curriculum is on average 2 years earlier than the international average.
(d) The textbooks in HK focus much of their attention on students' performance of "knowing" and "using routine procedures".
(e) A "canonical" curriculum is usually stipulated by the governments in Asian countries and is followed closely in schools.
(f) East Asian countries put a lot of emphasis on textbooks; by contrast, Western countries are more flexible in their use of textbooks.
(g) Tracking for mathematics teaching is common, and there are various ways of implementing tracking in different countries.
(h) HK is probably the place with the least flexibility and choice in its mathematics curriculum.

The results of TIMSS, which are relevant to the theme of the study, are also summarized:

(a) HK students came fourth both in the 26 countries in grade four and the 41 countries in grade eight. They performed very well in routine problem solving, not so well in solving exploratory problems, and significantly worse in the TIMSS Performance Assessment, where students were required to conduct hands-on activities.

[8] The summaries are abridged from Chapter 3 of the final report of the Ad hoc Committee, namely *Report on Holistic Review of the Mathematics Curriculum*.

(b) Students in HK, like their counterparts in the rest of the TIMSS countries, found mathematics important, but they did not particularly like mathematics.
(c) Contrary to the common belief that students in East Asian countries attribute success more to hard work than to natural talent or ability, and that they attach a lot of importance to memorization, the TIMSS results indicate that students do not totally support these stereotypes. Teachers in HK however did not tend to believe in natural talent.
(d) Students in HK did not think that they did well in mathematics and in general, girls had a lower perception of their ability than boys.
(e) Compared to their counterparts elsewhere, HK students spent more out of school time doing mathematics homework, studying mathematics or attending extra mathematics lessons, especially at the primary school level.

The results show that HK students did extremely well in the TIMSS mathematics tests, but some students did not display the corresponding level of positive attitudes towards mathematics and some lacked confidence in doing mathematics.

Research Study 2 – An Analysis of the Views of Various Sectors on the Mathematics Curriculum

The main findings are:

(a) Both students and parents showed high regard for mathematics.
(b) Different stakeholders held a positive view of the mathematics curriculum.
(c) Mathematics education should address a wider objective. HOTs should be addressed and teaching should provoke student thinking.
(d) The interest of students has to be maintained.
(e) The curriculum should be re-designed with epistemological and pedagogical considerations, so as to strengthen thinking and conceptual understanding.
(f) The problem of learner differences has to be addressed, including curriculum differentiation at senior secondary level.
(g) The idea of core and extended curriculum is worth further exploration.
(h) Continuation at all levels should also be secured. Teachers at various learning stages should have knowledge of the curriculum of other learning stages.
(i) Assessment and examination pressure should be carefully handled.
(j) The teacher is the key person in curriculum reform and he/she needs guidance and support on various issues including use of IT, enhancement of process abilities and curriculum tailoring.
(k) Pre-service and in-service teacher education should be strengthened.

(l) Collegiate exchange in the field should be promoted.

(m) Different stakeholders should be well informed of future curriculum changes, so that they provide support.

(n) The workload of teachers should be carefully considered.

Appendix 2

List of Tryout Topics/Teaching Strategies Conducted since 1997

Appendix 2

List of Tryout Topics/Teaching Strategies Conducted since 1997

Key Stages	Topics/Teaching Strategies
1	An Investigation of the Composition of Numbers up to 18 Developing an Understanding of Fractions
2	Squares and Square Roots Equivalent Fractions Simple Problems on Fractions Simple Problems on the Multiplication of Fractions Fractions, Decimals, Percentages and Recurring Decimals (Using Calculators) Recurring Decimals Multiplication of Decimals Multiplying Decimals by Decimals Chance : Its Meaning and Applications Chance Developing an Understanding of Chance Speed
3	Geometry – Transformation and Symmetry 3-Dimensional Solids Estimation Data Handling – Construction and Interpretation of Graphs Using IT in the Teaching and Learning of Geometry Using IT in Teaching & Learning Mathematics High-Order Thinking Skills

4	Data Handling – Simple Statistical Surveys Further Applications Using IT in the Teaching and Learning of Algebra Using IT in Teaching & Learning Mathematics

Appendix 2

Appendix 3

An Overview of Learning Targets of the Mathematics Curriculum

An Overview of Learning Targets of the Mathematics Curriculum

General Aims

To enable students to cope confidently with the mathematics needed in their future studies, workplaces or daily life in a technological and information-rich society, so that each student is ready for lifelong learning, the curriculum aims at developing in students:

(a) the ability to conceptualize, inquire and reason mathematically, and to use mathematics to formulate and solve problems in daily life as well as in mathematical contexts and other disciplines;

(b) the ability to communicate with others and express their views clearly and logically in mathematical language;

(c) the ability to manipulate numbers, symbols and other mathematical objects;

(d) number sense, symbol sense, spatial sense and a sense of measurement as well as the capability of appreciating structures and patterns;

(e) a positive attitude towards mathematics learning and the capability of appreciating the aesthetic nature and cultural aspect of mathematics.

Knowledge and Skills

The Learning Targets for Key Stage 1 (P1-P3)				
Number and Algebra Dimensions		Measures, Shape and Space Dimensions		Data Handling Dimension
Number	Algebra	Shape & Space	Measures	Data Handling
• To understand and manipulate whole numbers • To understand simple fractions • To examine the reasonableness of results • To formulate and solve simple problems involving numbers	The ALGEBRA Dimension is not included at this key stage.	• To identify, describe and group lines, angles, 2-dimensional & 3-dimensional shapes • To recognize intuitively the elementary properties of 3-dimensional shapes • To recognize the properties of 2-dimensional shapes • To make 2-dimensional and 3-dimensional shapes from given information • To recognize and appreciate shapes • To identify the four directions	• To choose and use a variety of non-standard units to record results in basic measuring activities • To understand the need to use standard units of measurement • To select appropriate measuring tools and standard units of measurement • To integrate knowledge of Number, Measures, Shape & Space to solve simple problems in measurement	• To collect, compare and group discrete statistical data according to given criteria • To construct and interpret simple statistical graphs showing relations among data • To formulate and solve simple problems arising from collected data and constructed graphs

Appendix 3

The Learning Targets for Key Stage 2 (P4-P6)				
Number and Algebra Dimensions		Measures, Shape and Space Dimensions		Data Handling Dimension
Number	Algebra	Shape & Space	Measures	Data Handling
• To understand whole numbers, fractions, decimals, percentages and the relations among them • To manipulate numbers and examine the reasonableness of results • To formulate and solve problems involving numbers	• To use symbols to represent unknown numbers • To communicate simple mathematical facts and relations using symbols • To formulate and solve simple problems and examine the results	• To understand the properties of 2-dimensional and 3-dimensional shapes • To group and make 2-dimensional and 3-dimensional shapes • To identify the eight compass points	• To choose and use a variety of non-standard and standard units to record results in various measuring activities • To select and justify appropriate measuring tools and standard units of measurement • To recognize the degree of accuracy and the approximate nature of measurement • To inquire and use simple measurement formulae • To integrate knowledge of Number, Measures, Shape & Space to formulate and solve simple problems in measurement	• To understand the criteria for organizing and grouping discrete statistical data • To apply simple arithmetic and appropriate scales in constructing and interpreting more complex statistical graphs • To show relationships among data using a variety of statistical and graphical representations • To recognize relations and patterns from graphs • To formulate and solve problems arising from collected data and constructed graphs

Appendix 3

The Learning Targets for Key Stage 3 (S1-S3)		
Number and Algebra Dimension	Measures, Shape and Space Dimension	Data Handling Dimension
• To experience rational and irrational numbers • To develop various strategies in using numbers to formulate and solve problems, and to examine results; • To develop and refine strategies for estimating • To extend the use of algebraic symbols in communicating mathematical ideas • To explore and describe patterns of sequences of numbers using algebraic symbols • To interpret simple algebraic relations from numerical, symbolic and graphical perspectives • To manipulate algebraic expressions and relations; and apply the knowledge and skills to formulate and solve simple problems and to examine results • To interconnect the knowledge and skills in various Learning Dimensions to solve problems	• To understand the nature of measurement and be aware of the issues about precision and accuracy • To apply a variety of techniques, tools and formulas for measurements and solving mensuration problems • To explore and visualize geometric properties of 2-dimensional and 3-dimensional objects intuitively • To use inductive reasoning, deductive reasoning and an analytic approach to study the properties of 2-dimensional rectilinear shapes • To formulate and write simple geometric proofs involving 2-dimensional rectilinear shapes with appropriate symbols, terminology and reasons • To inquire, describe and represent geometric knowledge in 2-dimensional figures using numeric and algebraic relations; • To inquire geometric knowledge in 2-dimensional space using trigonometric relations • To interconnect the knowledge and skills in various Learning Dimensions to solve problems	• To understand the criteria for organizing discrete and continuous statistical data • To choose and construct appropriate statistical diagrams and graphs to represent given data and interpret them • To compute, interpret and select the appropriate measure to describe the central tendency of a set of data • To judge the appropriateness of the methods used in handling statistical data • To understand the notion of probability and handle simple probability problems by listing and drawing diagrams

Appendix 3

The Learning Targets for Key Stage 4 (S4-S5)		
Number and Algebra Dimension	Measures, Shape and Space Dimension	Data Handling Dimension
• To recognize different types of numbers • To investigate and describe relationships between quantities using algebraic symbols and relations • To generalize and describe patterns of sequences of numbers using algebraic symbols; and apply the results to solve problems • To interpret more complex algebraic relations from numerical, symbolic and graphical perspectives • To manipulate more complex algebraic expressions and relations, and apply the knowledge and skills to formulate and solve a variety of practical problems and justify the validity of results • To interconnect the knowledge and skills in various Learning Dimensions to solve problems	• To use and select inductive reasoning, deductive reasoning or an analytic approach to study the properties of 2-dimensional shapes • To formulate and write geometric proofs involving 2-dimensional shapes with appropriate symbols, terminology and reasons • To inquire, describe and represent geometric knowledge in 2-dimensional space using algebraic relations • To inquire, describe and represent geometric knowledge in 2-dimensional and 3-dimensional space using trigonometric functions • To interconnect the knowledge and skills in various Learning Dimensions to solve problems	• To understand and compute the measures of dispersion • To select and use measures of central tendency and dispersion to compare data sets; • To investigate and judge the validity of arguments derived from a data set • To formulate and solve further probability problems by applying simple laws • To integrate knowledge in statistics and probability to solve real life problems

Appendix 4

Developing Generic Skills in the Mathematics Education Key Learning Area

Appendix 4

Developing Generic Skills in the Mathematics Education Key Learning Areas

Collaboration Skills

Problem solving, planning and making decisions in a small group require the necessary collaboration skills, namely the skills of listening, appreciation, communication, negotiation, making compromises, asserting leadership, making judgement, as well as influencing and motivating others. Learners with these skills will be able to effectively engage in tasks and teamwork as well as working with others. Ultimately, learners will be able to form relationships that are mutually beneficial.

(The expected achievements of the learners in this type of generic skills cannot be suitably classified according to key learning stages)

Descriptors of Expected Achievements across the School Curriculum	Exemplars of Implementation in Mathematics Education
Understanding working relationships Learners will learn to • clarify and accept various roles and responsibilities of individual members in a team and be willing to follow team rules • recognize that individuals as well as the team have to take the consequences for their own actions	Learners 1. share responsibilities and understand the roles of individual members in doing mathematical group work like collecting data, measuring objects and presenting projects 2. understand and accept that members with different cultural backgrounds may have different interpretations of a mathematical problem (e.g. analyzing statistical data) 3. accept and follow the group decision in doing mathematical group work
Developing attitudes which contribute to good working relationships Learners will learn to • be open and responsive to others' ideas; appreciate, encourage and support the ideas and efforts of others • be active in discussing and posing questions to others, as well as in exchanging, asserting, defending and rethinking ideas • recognize and avoid stereotyping; withhold premature judgement until the facts are known • be willing to adjust their own behaviour to fit the dynamics of various groups and situations	Learners 1. discuss and exchange ideas openly with others in completing tasks and solving mathematical problems 2. be patient and listen to others in the discussion of mathematical problems like experience-sharing in the processes of investigating number patterns or formulating proofs of geometric problems 3. value the contributions of others in accomplishing mathematical tasks or solving mathematical problems together 4. appreciate different solutions to mathematical problems presented by others, for example, in using different approaches to proving mathematical theorems 5. participate actively and pose questions in clarifying one's arguments in solving mathematical problems, for example, in the discussion of strategies to be adopted in investigating practical statistical problems

Appendix 4

Descriptors of Expected Achievements across the School Curriculum	Exemplars of Implementation in Mathematics Education
Achieving effective working relationships Learners will learn to ♦ select a strategy and plan cooperatively to complete a task in a team ♦ understand the strengths and weaknesses of members and build on the strengths to maximize the potential of the team ♦ liaise, negotiate and compromise with others ♦ reflect on and evaluate the group work strategy and make necessary adjustments	Learners 1. share experience in solving mathematical problems and select cooperatively a suitable strategy to solve a mathematical problem 2. clarify their arguments objectively and rationally in solving mathematical problems, for example, in examining the appropriateness of the strategy adopted in solving mathematical problems 3. liaise, negotiate and compromise with others in selecting a suitable strategy for solving a mathematical problem (e.g. use a synthetic or analytic approach in solving a geometrical problem) 4. make adjustment, if necessary, to the strategy adopted in solving mathematical problems (e.g. is it necessary to solve a given quadratic equation to prove that it has no real roots?)

Communication Skills

Communication is a dynamic and ongoing process in which two or more people interact in order to achieve a desired outcome or goal. In learning to communicate effectively, learners should learn to speak, listen, read and write effectively. They should learn to select the most appropriate means to convey a message in accordance with the purpose and context of the communication. They should use accurate and relevant information and organize it systematically and coherently for their audience. They should also evaluate the effectiveness of their communication and identify areas of improvement for action.

Descriptors of Expected Achievements across the School Curriculum	**Exemplars of Implementation in Mathematics Education**
Key Stage One (Junior Primary) Learners will learn to • comprehend and act appropriately on spoken instructions • use clear and appropriate means of communication, both verbal and non-verbal, to express meaning and feelings • read and write simple texts	Learners 1. describe objects such as cubes and prisms orally with simple and appropriate mathematical terms (e.g. a cube has six faces) 2. interpret drawings, tables, graphs (e.g. pictograms) and symbols (e.g. ＋, －, ×) 3. present findings with drawings and symbols 4. present data with tables and graphs (e.g. block graphs) 5. describe drawings and symbols in plain language (e.g. 2＋3 as 2 plus 3) 6. express simple daily-life problems in mathematical language (e.g. use symbols like $2×3 and graphs like bar graphs)
Key Stage Two (Senior Primary) Learners will learn to • comprehend and respond to different types of texts • use spoken, written, graphic and other non-verbal means of expression to convey information and opinions, and to explain ideas • work and negotiate with others to develop ideas and achieve goals	Learners 1. interpret drawings, symbols (e.g. %), tables and graphs (e.g. broken line graphs) 2. describe and explain findings / results / data of mathematical tasks in both oral and written forms (e.g. the average score of a student's performance in a test, the favorite fruit) 3. present results of tasks with appropriate drawings and symbols 4. present data with tables, charts and graphs (e.g. broken line graph, straight line graph) 5. describe and analyze data 6. present solutions of problems logically (e.g. use of "=" properly) 7. express simple problems in mathematical language (e.g. the percentage of discount is 10%) 8. discuss with others in accomplishing tasks (such as projects)

Appendix 4

Descriptors of Expected Achievements across the School Curriculum	Exemplars of Implementation in Mathematics Education
Key Stage Three (Junior Secondary) Learners will learn to • understand, analyze, evaluate and respond to a range of different types of texts • use appropriate language and / or other forms of communication to present information and different points of view, and to express feelings • reflect and improve on the effectiveness of their own communication • work and negotiate with others to solve problems and accomplish tasks	Learners 1. interpret numeric, symbolic and graphical presentations 2. describe findings or explain conjectures in both oral and written forms using mathematical language (e.g. the two triangles are congruent) 3. choose appropriate statistical diagrams / graphs to present given data and use appropriate mathematical terminology or symbols in explaining ideas 4. formulate and write simple geometric proofs involving 2-D rectilinear shapes with appropriate symbols, terminology and reasons 5. interpret and respond appropriately to others' mathematical arguments in both oral and written forms 6. distinguish the difference between the language used in a mathematical context and daily life (e.g. rate, similar) 7. use mathematical language including graphs, figures and symbols to analyze and present possible solutions to a problem and discuss with others
Key Stage Four (Senior Secondary) Learners will learn to • listen and read critically, and speak and write fluently for a range of purposes and audiences • use appropriate means of communication to inform, persuade, argue and entertain and achieve expected outcomes • critically evaluate the effectiveness of their communication • resolve conflicts and solve problems with others to accomplish tasks	In addition to points 1-6 in KS3, point 4 is modified as follows: Learners 4. formulate and write geometric proofs involving more complex 2-D shapes with appropriate symbols, terminology and reasons Further points include: Learners 8. investigate and judge the validity of arguments presented in statistical reports from various sources, including those from the media (e.g. select appropriate samplings for statistical surveys) 9. resolve problems with others in accomplishing tasks such as projects

Appendix 4

Creativity

A brief description: Creativity is an important but elusive concept. It has been defined in a variety of ways. Some people define it as an ability to produce original ideas and solve problems, others see it as a process, and yet others take it as certain personal qualities. In fact, creativity is a complex and multifaceted construct. Within the individual, creative behaviour is the result of a complex of cognitive skills / abilities, personality factors, motivation, strategies, and metacognitive skills. Person's creative performance may not correspond to his / her developmental stages.

General Principles: Although the demanding process of teaching for creativity is hard to make routine, some principles apply in general. To develop students' creativity, we ask them to go beyond the given information, allow them time to think, strengthen their creative abilities, reward their creative efforts, value their creative attributes, teach them creative thinking techniques and the Creative Problem Solving model, and create a climate conducive to creativity[1]. These principles can be employed in all key learning areas (KLAs).

(The expected achievements of the learners in this type of generic skills cannot be suitably classified according to key learning stages)

Descriptors of Expected Achievements across the School Curriculum	**Exemplars of Implementation in Mathematics Education**
Learners will learn to • **strengthen creative abilities**: fluency[2], flexibility[3], originality[4], elaboration[5], sensitivity to problems[6], problem defining[7], visualization[8], imagination, analogical thinking[9], analysis, synthesis, evaluation, transformation[10], intuition, logical thinking, etc. • **develop creative attitudes and attributes**: imagination, curiosity, self-confidence, independent judgement, persistence and commitment, tolerance for ambiguity, openness to new and unusual ideas / methods / approaches, deferment of judgement, adaptability, willingness to take sensible risks, etc. • **use and apply the Creative Problem Solving (CPS) Model and creative thinking techniques**: brainstorming, 6W thinking technique, 6 hats method, attribute listing[11], idea checklists, synectics[12], mind mapping, etc.	Learners 1. create geometric patterns with different shapes and tell stories with given mathematical sentences 2. devise their own way / strategy in solving problems such as different solutions to a plane geometry problem 3. adopt different approaches to a task or problem, such as proving a geometrical theorem using a synthetic or an analytical approach 4. pose related problems such as "Can triangles other than equilateral triangles be used in tessellation?" and "Will the same relationship $a^2 + b^2 = c^2$ in Pythagoras' Theorem still hold if the triangle is not right-angled?" 5. formulate hypotheses such as that the value of a fraction decreases as the denominator increases if the numerator is kept constant 6. be persistent when solving problems 7. be imaginative in visualizing 3-D shapes 8. be open-minded to new methods and approaches in accomplishing tasks and solving problems, such as using a synthetic or an analytic approach in solving geometrical problems 9. use and apply the technique of synectics to relate different given information, and utilize analogies to help analyze problems, such as deducing the formula of the volume of a cylinder from that for a prism

Appendix 4

Notes:

1. Climate conducive to creativity: Respecting the novel and unusual, providing challenges, appreciating individuality and openness, encouraging open discussion, absence of conflicts, allowing time for thinking, encouraging confidence and a willingness to take risks, appreciating and supporting new ideas, etc.
2. Fluency: The ability to produce many ideas in response to an open-ended problem, question or task.
3. Flexibility: The ability to take different approaches to a task or problem, to think of ideas in different categories, or to view a situation from several perspectives.
4. Originality: Uniqueness, nonconformity in thought and action.
5. Elaboration: The ability to add details to a given idea, such as to develop, embellish, and implement the idea.
6. Sensitivity to problems: The ability to identify problems, list out difficulties, detect missing information, and ask good questions.
7. Problem defining: The capability to 1) identify the "real" problem, 2) isolate the important aspects of a problem, 3) clarify and simplify a problem, 4) identify subproblems, 5) propose alternative problem definitions, and 6) define a problem broadly.
8. Visualization: The ability to fantasize and imagine, "see" things in the "mind's eye" and mentally manipulate images and ideas.
9. Analogical thinking: The ability to borrow ideas from one context and use them in another; or the ability to borrow the solution to a problem and transfer it to another.
10. Transformation: The ability to adapt something to a new use, to "see" new meanings, implications, and applications, or to change an object or idea into another creatively.
11. Attribute listing: A creative thinking technique that involves listing out all the important characteristics of an item and suggesting possible changes or improvements in the various attributes.
12. Synectics: The joining together of apparently unrelated elements. This technique utilizes analogies and metaphors to help the thinker analyze problems and form different viewpoints.

Appendix 4

Critical Thinking Skills

Critical Thinking is drawing out meaning from given data or statements. It is concerned with the accuracy of given statements. It aims at generating and evaluating arguments. Critical thinking is the questioning and inquiry we engage in to judge what to and what not to believe.

Descriptors of Expected Achievements across the School Curriculum	Exemplars of Implementation in Mathematics Education
Key Stage One (Junior Primary) Learners will learn to • extract, classify and organize information from a source • identify and express main ideas, problems or central issues • understand straightforward cause-and-effect relationships • distinguish between obvious fact and opinion • recognize obvious stereotypes, assumptions, inconsistencies and contradictions • formulate questions, make predictions / estimations and hypotheses • draw simple but logical conclusions not contradictory to given evidence and data	Learners 1. sort objects using various criteria such as shapes and sizes 2. choose the right tools to measure objects such as using measuring tapes to measure the "circumference" of a head 3. reason inductively such as when exploring the commutative property of addition 4. check the reasonableness of the answer to a problem (e.g. the number of apples eaten by a boy per day found in one problem is too large to be realistic)

Descriptors of Expected Achievements across the School Curriculum	Exemplars of Implementation in Mathematics Education
Key Stage Two (Senior Primary) Learners will learn to • understand and make deductions / inferences from sources • cross reference other sources to determine the reliability of a source • understand the concepts of relevance and irrelevance • distinguish fact and opinion as well as source and evidence • question obvious bias, propaganda, omissions, and the obvious fallacies • formulate appropriate questions, make reasonable predictions and hypotheses • draw logical conclusions based on adequate data and evidence, and make predictions about consequences	Learners 1. categorize information using various criteria such as properties of quadrilaterals 2. choose appropriate methods and units to measure objects, such as using the method of displacement to measure the volumes of irregular objects 3. reason inductively such as when exploring the formula for the area of a rectangle 4. check the reasonableness of the solution to a problem (e.g. the steps for solving a problem are unreasonably complicated)
Key Stage Three (Junior Secondary) Learners will learn to • compare different sources, note contrasts and similarities, and determine their reliability • distinguish fact, opinion and reasoned judgment • be aware that value orientations and ideologies would affect the perspective of a source • recognize and challenge stereotypes, inconsistencies, emotional factors, and propaganda • draw and test conclusions as well as hypotheses, identify reasonable alternatives and predict probable consequences	Learners 1. use inductive and deductive reasoning to study the properties of geometric shapes, such as when proving "the sum of the interior angles of a polygon" and "the base angles of an isosceles triangle" 2. generalize observations in symbolic forms from concrete experiences, such as when generalizing the index laws from observing several examples in numbers 3. judge whether the information given about a problem is relevant or not (e.g. extraneous data given in a geometrical problem) 4. examine the reasonableness of the solution to a problem and evaluate the strategy adopted (e.g. evaluate the effectiveness of using the graphical method to solve simple linear equations)

Appendix 4

Descriptors of Expected Achievements across the School Curriculum	Exemplars of Implementation in Mathematics Education
Key Stage Four (Senior Secondary) Learners will learn to • distinguish real and stated issues, false and accurate images, and relevant and irrelevant evidence • recognize and challenge subtle consistencies and inconsistencies, unstated fundamental assumptions, permeating value orientations and ideologies • distinguish among sophisticated fact, opinion and reasoned judgment • be aware that the selection and deployment of information / facts is affected by personal perspective • draw warranted conclusions, predict and assess probable consequences and make reasoned judgment in reading, writing, and speech	Learners 1. use and select inductive reasoning, deductive reasoning or analytic approach to study the properties of various shapes including 2-D shapes such as proving "the angles in the same segment", finding the general equation of a circle 2. generalize observations in symbolic forms from concrete experiences such as finding the general term of an arithmetic sequence 3. judge the relevance of information given to a problem (e.g. the relevance of the mean of a given set of data given in a problem of computing the range) 4. judge the validity of arguments presented in various reports including statistical reports (e.g. the abuse of statistical data) 5. examine the reasonableness of the solution to a problem and evaluate the strategy adopted (e.g. the strategy used to solve a geometrical problem involving circles)

Appendix 4

Information Technology Skills

IT skills are the ability to use IT to seek, absorb, analyze, manage and present information critically and intelligently. In addition, IT will motivate and empower our learners to learn at their own pace and help them develop habits of self-learning, which will benefit them for life.

Descriptors of Expected Achievements across the School Curriculum	Exemplars of Implementation in Mathematics Education
Key Stage One (Junior Primary) Learners will learn to • operate computers in schools • input Chinese characters with a handwriting recognition device • use multimedia resources to support learning with the help of teachers • communicate and handle information with IT tools in learning activities	Learners 1. use suitable software to investigate number patterns and properties of numbers (e.g. odd, even, ascending, descending) 2. use suitable software to create and explore geometric patterns (e.g. squares, rectangles, triangles)
Key Stage Two (Senior Primary) Learners will learn to • use a number of software packages for different purposes • input Chinese characters with devices and the aid of an input method • access information via computer networks and other media • process information using IT tools	Learners 1. use suitable software to investigate the properties of shapes, draw and create geometric patterns (e.g. draw squares, rectangles, triangles, circles and create geometric patterns with these figures.) 2. use a spreadsheet to record data and create graphs for doing statistical projects (e.g. input data in a spreadsheet and present the data with line graphs, bar charts) 3. use the information obtained through Internet / Intranet for self-learning and doing projects (e.g. symmetry)

Appendix 4

Descriptors of Expected Achievements across the School Curriculum	Exemplars of Implementation in Mathematics Education
Key Stage Three (Junior Secondary) Learners will learn to • use appropriate IT tools to facilitate learning • use IT tools and strategies for processing and presenting information • communicate with others via e-mails • verify and evaluate the accuracy and reliability of information	Learners 1. use scientific calculators / graphing calculators for various computational and exploratory activities (e.g. input data and create statistical graphs; draw straight lines and explore their relationship with slope) 2. use suitable software to explore relations of numbers (e.g. number patterns), algebraic formula (e.g. formulae of area and volume) and graphical representations (e.g. pie charts and straight lines) 3. use suitable software to construct / explore appropriate statistical diagrams / graphs (e.g. bar charts, pie charts, line graphs) to represent given data; to find simple statistical measures (e.g. mean, mode) and to explore the meaning of experimental probability (e.g. simulation of tossing coins) 4. use geometry software to explore properties of 2-D rectilinear geometric figures dynamically (e.g. the relationship among the angles or sides of a parallelogram); to explore and visualize geometric properties of 2-D and 3-D figures intuitively (e.g. transformation and symmetry) 5. use the information obtained through Internet / Intranet for self-learning and doing projects (e.g. statistical projects, projects on the development of mathematics in China, mathematicians) 6. judge the appropriateness of using IT in solving mathematical problems (e.g. quicker to calculate $2\sin 30°$ mentally)

Descriptors of Expected Achievements across the School Curriculum	Exemplars of Implementation in Mathematics Education
Key Stage Four (Senior Secondary) Learners will learn to • improve self productivity • use and analyze information • produce multimedia presentations • integrate the uses of a wide range of IT tools to fulfill specific purposes • select and apply appropriate IT tools in different aspects of study, like research, etc	Learners In addition to the points in KS3, some modifications of points 2, 4 and 5 are as follows: Learners 2. use spreadsheets or other appropriate Computer Algebraic Systems to investigate and describe relationships between quantities using algebraic symbols and relations (e.g. find the sum of an arithmetic sequence or a geometric sequence); to manipulate more complex algebraic expressions and relations in solving problems (e.g. the discriminant of a quadratic equation and the sum of a geometric sequence) 4. use geometry software to explore the properties of more complex 2-D geometric figures dynamically (e.g. the relationship of angles in a cyclic quadrilateral); to inquire and describe geometric knowledge in 2-D and 3-D space using trigonometric functions, (e.g. draw 2-D or 3-D figures, visualize the figures from different angles, express the lengths of the sides using trigonometric functions) 5. use Internet / Intranet to obtain statistical data and information for performing statistical analysis; to obtain statistical reports for discussion of the methods of the investigation and the validity of arguments derived from the investigation (e.g. discuss the method of collecting data, size of the set of data, source of the data, ways of presenting the data, use and abuse of statistical graphs)

Appendix 4

Numeracy Skills

Numeracy skills include the ability to perform basic computations, to use basic mathematical concepts in practical situations, to make reasonable estimates, to understand graphs, charts and numerical concepts in languages, to manage data, to handle money and do stock inventories.

Descriptors of Expected Achievements across the School Curriculum	Exemplars of Implementation in Mathematics Education
Key Stage One (Junior Primary) Learners will learn to • perform basic computations • recognize and describe shape, position and direction • develop an intuitive knowledge of measurement and measuring units, and use appropriate tools for measurements e.g. ruler, thermometer • formulate and solve simple problems arising from collected data and constructed graphs • read and use simple quantitative information	Learners 1. describe the number of objects with natural numbers 2. perform basic computations involving whole numbers properly 3. use non-standard and standard measuring tools in comparing measures of different objects 4. recognize basic directions: north, south, east and west 5. recognize and describe 2D-shapes such as triangles and quadrilaterals and 3-D shapes such as pyramids and prisms 6. read simple statistical graphs and charts such as block graphs and simple pictograms
Key Stage Two (Senior Primary) Learners will learn to • perform numerical computations, calculate mentally and provide quick estimates of the accuracy of a calculation • understand intuitively the properties of shape, position and direction • extend measurement skills to concept areas such as volume • collect, process, present and evaluate quantitative information • use mathematical concepts to solve simple real-life problems	Learners 1. choose the correct forms of numbers in presenting information like using percentages to tell the discount obtained 2. perform numerical computations to solve daily-life problems and evaluate their own work 3. apply formulae to find measures of simple 2-D shapes such as the area of a triangle 4. describe the measure with appropriate units like using m^2 to measure the area of a courtyard while using cm^2 to measure the area of a sheet of paper 5. organize simple data and interpret simple statistical graphs in various daily-life situations 6. recognize the 8 compass points like north-east, south-west, etc. 7. use equations to solve simple problems

Descriptors of Expected Achievements across the School Curriculum	Exemplars of Implementation in Mathematics Education
Key Stage Three (Junior Secondary) Learners will learn to • perform numerical manipulations and quick estimates of the accuracy of a calculation • understand properties of shape, position, direction and movement • apply formulae or choose the appropriate tools and strategies to find measures and note the approximate nature of measurement • use appropriate tools and strategies for collecting, processing and presenting quantitative information • estimate risks and chances through the use of elementary probability • solve real-life experiences utilizing quantitative information	Learners 1. manipulate numbers, algebraic symbols, trigonometric relations, etc. to solve real-life problems 2. apply numerical estimation strategies and estimation strategies in measurement to various real-life situations such as the number of significant figures 3. apply formulae to find measures of 2-D and 3-D shapes such as the area of a sector and the volume of a sphere 4. apply spatial concepts in real-life situations like the angle of elevation and bearings 5. apply simple ideas of data handling to understand statistical data and graphs in various sources from real-life experience 6. apply simple ideas of probability in various real-life situations such as the fairness of games
Key Stage Four (Senior Secondary) Learners will learn to • solve problems involving numbers and symbols by using quantitative evidence and appropriate devices • evaluate the appropriateness of tools and strategies for collecting, processing and presenting quantitative information • adapt to new mathematical demands in various circumstances as needed • use quantitative information for personal organization and planning, and for understanding social problems	In addition to points in KS3, further elaborations on points 1 and 5 are: Learners 1. use functions to describe real-life phenomena and solve real-life problems such as population growth, sound intensity, etc. 5. apply statistical measures to analyze/interpret data, reports and information in various real-life sources (e.g. using the idea of sampling in studying bird flu problems) Further points include Learners 7. integrate mathematical knowledge and skills to solve more complex real-life problems (e.g. integrating the properties of space and trigonometric ratios to solve navigation problems) 8. solve real-life problems by selecting knowledge from different mathematical disciplines such as using the idea of linear programming to maximize the profit of a production under given constraints

Appendix 4

Problem Solving Skills

Problem solving involves using thinking skills to resolve a difficulty. It assembles facts about the problem and determines the best course of action.

Descriptors of Expected Achievements across the School Curriculum	Exemplars of Implementation in Mathematics Education
Key Stage One (Junior Primary) Learners will learn to • develop ideas about the problem and identify sources of information and help • identify, under guidance, different ways of tackling the problem • choose and implement a solution plan, using support and advice given • follow the given step-by-step methods to check and describe the outcomes	Learners 1. use simple methods of solving problems (e.g. using addition to find the sum of money used in buying goods) 2. adopt various ways of solving problems (e.g. using drawing and manipulative to do addition) 3. solve problems by choosing the correct given data (e.g. choosing the correct combination of coins for one exact bus fare)
Key Stage Two (Senior Primary) Learners will learn to • identify the problem and describe its main features • propose alternative courses of action for solving it • plan and try out the selected option, obtain support and make changes when needed • develop an appropriate method to measure the outcomes and examine the approach chosen	Learners 1. use different approaches to solving problems (e.g. finding the area of a shape by means of counting the number of squares, dissecting the shape into parts or using formulae) 2. make use of various tools in solving problems (e.g. measuring tapes and calculators) 3. choose the correct given data and relevant information to solve problems (e.g. identify the correct height and base in finding the area of a triangle) 4. use past experience to solve new problems, for example, by comparing the problems with some similar ones which have been solved before (e.g. comparing the ways of finding the area of a right-angled triangle and that of a general one) 5. adopt various ways of solving problems such as using tables and formulae (e.g. formulae for perimeter, area, etc.)

Descriptors of Expected Achievements across the School Curriculum	Exemplars of Implementation in Mathematics Education
Key Stage Three (Junior Secondary) Learners will learn to • explore the problem and identify the issue(s) at stake • suggest and compare the possible outcomes of each alternative course of action and justify the option selected • execute the planned strategy, monitor progress and revise the approach when necessary • evaluate against established criteria the quality of outcomes, and review the effectiveness of the solution process	Learners 1. understand a given problem; devise an appropriate plan for solving problems (e.g. geometrical problem) and justify or evaluate the solution presented 2. appreciate and pursue a better strategy for solving problems (e.g. by comparing different strategies for solving geometrical problems and comparing different data sets with means and medians) 3. judge and evaluate arguments of their own or others in presenting a solution to a mathematical problem (e.g. explain why mode is a preferable measure of the most popular size of shoes in a market) 4. formulate the mathematical solution in tackling a real-life problem (e.g. finding the area of a layout plan of a flat)
Key Stage Four (Senior Secondary) Learners will learn to • recognize the complexity of the problem and search for appropriate information required to solve it • formulate feasible strategies to achieve optimal results, considering both long term as well as short term objectives • monitor and critically reflect on the progress in solving the problem • evaluate the overall strategy and anticipate possible future problems related to the solution	Learners 1. understand a given problem and propose the method of approach (e.g. exploring the meaning of probability through various activities) 2. appreciate and formulate feasible strategies to achieve the solution of a problem (e.g. calculating probability by tabulation or by listing the possible outcomes) 3. judge and evaluate mathematical arguments (e.g. “Two triangles are congruent because their corresponding angles are equal.”) 4. monitor and analyze the given information critically (e.g. recognizing the dangers of misinterpreting statistical data)

Appendix 4

Self Management Skills

Self-management skills are essential for the building up of self-esteem and the accomplishment of goals. Learners who have mastered self-management skills understand their own feelings and preserve emotional stability. They are positive and proactive towards work. They set appropriate goals, make plans and initiate actions to achieve them. They manage time, money and other resources well. They are able to handle stress and tolerate ambiguities.

Learners will learn to

1. evaluate their own feelings, strengths, weaknesses, progress and objectives (self-assessment)
2. consider aspects of their performance, attitudes and behaviour in order to change or enhance future outcomes (self-reflection)
3. be confident in their own judgements, performance and capabilities (self-confidence)
4. make informed decisions and safe choices in reaching goals and carrying tasks, develop good habits and maintain a healthy life style (self-discipline)
5. work under unfamiliar, stressful or adverse conditions, accept changes and new ideas and be able to handle diversity and tolerate ambiguity (adaptability)
6. make decisions and initiate actions on their own and draw satisfaction from their own effort (self-motivation)
7. keep promises and fulfill obligations (sense of responsibility)
8. control their own emotions and impulses and maintain emotional balance (emotional stability)

(The expected achievements of the learners in this type of generic skills cannot be suitably classified according to key learning stages)

Appendix 4

Descriptors of Expected Achievements across the School Curriculum	Exemplars of Implementation in Mathematics Education
• Self assessment • Self reflection • Self confidence • Self discipline • Adaptability / Ability to Work with Diversity • Self motivation • Sense of Responsibility • Emotional Stability	Learners: 1. work neatly and tidily in accomplishing tasks (such as statistical projects) and doing mathematics problems (such as drawing geometric figures with rulers and pencils) 2. appreciate / accept and evaluate others' opinions in accomplishing tasks (such as different ways of collecting data) and solving problems (such as different strategies for solving quadratic equations and geometrical problems) 3. self-evaluate their own strengths and weaknesses in mathematics learning 4. should be responsible for their own work, for example, by checking the answers to problems 5. should be self-reliant, confident and willing to apply mathematics in solving problems independently 6. should be persistent in solving difficult / complicated problems 7. keep emotions under control when facing failure in solving problems 8. communicate their own feeling to others when facing failure in solving problems 9. set goals and priorities properly in mathematics learning

Appendix 4

Study Skills

Study skills are the basic techniques that help to improve the effectiveness and efficiency of learning. They are crucial to the development of the basic learning habits, abilities and attitudes of the learners that form the essential foundation for lifelong learning.

Descriptors of Expected Achievements across the School Curriculum	Exemplars of Implementation in Mathematics Education
Key Stage One (Junior Primary) Learners will learn to • identify the main points and ideas in different types of straightforward reading materials • use different forms of writing to present main ideas clearly • collect information from given sources, organize them into predetermined categories and analyze them according to preset guidelines • understand the need to set up a study plan and follow a given plan to meet short-term targets	Learners 1. understand concepts and do not learn only by rote memorization (e.g. pupils have to understand the concept of multiplication and build up the multiplication tables themselves instead of just memorizing the multiplication tables; understand the concept of perimeters and use various ways to find the perimeters of figures instead of using formulae only) 2. present simple problems in mathematical languages including symbols (like 1+2=3) and graphs (like pictograms) 3. use simple objects to build up mathematical concepts (e.g. use $1cm^3$ cubes or an abacus to grasp the concept of addition, subtraction; and use a pin board to grasp the concept of perimeter and area of rectangles)
Key Stage Two (Senior Primary) Learners will learn to • identify main lines of reasoning, skim materials to gain general ideas of content and scan text to obtain main points • use different forms and styles of writing for different purposes and present main ideas coherently in a given form and style of writing • locate required information from a variety of sources, organize them into self-defined categories and assess them for completeness, accuracy and relevance • develop short-term and intermediate study plans to meet targets and purposes of study identified by oneself	Learners 1. try to understand concepts and not to learn only by rote memorization (e.g. pupils have to understand the concepts and formulae of areas and volumes instead of memorizing the formulae in solving problems) 2. use more aids to build up mathematical concepts (e.g. use a calculator to learn number patterns) 3. present problems in mathematical languages including symbols (like simple algebraic equations) and graphs (like bar charts) 4. use diagrams, pictures and charts to help understand mathematical concepts, for example, use travel graphs to understand the idea of speed 5. learn from mistakes made in solving mathematical problems in homework exercises, tests, etc

Appendix 4

Descriptors of Expected Achievements across the School Curriculum	Exemplars of Implementation in Mathematics Education
Key Stage Three (Junior Secondary) Learners will learn to • identify accurately complex lines of reasoning and hidden ideas and distinguish facts from opinions • select an appropriate form and style of writing for a specific purpose and develop a writing strategy for organizing ideas and information clearly and coherently • define purposes of collecting information, critically investigate sources to distil relevant information and evaluate the quality and validity of information • review and revise study plans developed for short-term, intermediate and long-term targets to meet new demands and to improve study performance	Learners 1. understand and do not only memorize by rote the meanings of different forms of mathematical objects, concepts and principles 2. generalize observations to a higher-level of abstraction for better memorization and for transferring strategies to solving a wider range of problems 3. learn from mistakes made in solving mathematical problems in homework exercises, tests, etc. 4. identify key similarities and differences from working among various types of mathematical problems 5. search and select information from various sources including libraries, reference books, Internet, etc. 6. use diagrams, pictures and charts to help understand mathematical concepts, for example, by using charts to present the hierarchy of the real number system
Key Stage Four (Senior Secondary) Learners will learn to • evaluate key ideas, opinions and arguments identified from reading materials and synthesize them to construct and develop their own interpretation and reflections • assess their own writing strategies to ensure relevant information, ideas and arguments are structured and presented in a logical sequence and the writing is in an appropriate form and style • explore alternative lines of inquiry, refine and integrate information into specific formats and evaluate an overall strategy for refinement and new requirements • evaluate an overall strategy for effectiveness and quality and adapt the strategy and seek alternatives as necessary, based on reflections and feedback	In addition to points KS3, learners 7. read mathematical problems extensively and independently 8. read and understand longer symbolic arguments including geometric proofs 9. evaluate their own arguments in completing tasks and problems, to ensure relevant information and ideas are structured and presented logically

Appendix 4

Appendix 5

Related Values and Attitudes

Appendix 5

Related Values and Attitudes

Besides knowledge and skills, the development of positive values and attitudes is also important in mathematics education. These values and attitudes permeate the mathematics curriculum in different learning dimensions and key stages of learning and have been incorporated into the learning objectives of the revised primary and secondary curricula. The following learning objectives illustrate how the revised mathematics curricula relate to the development of positive values and attitudes and aim at facilitating the planning of relevant learning experiences in the Mathematics KLA. These learning objectives, however, are neither exhaustive nor implying that the related values and attitudes should progress in the order stated. In fact, they can be realized at different key stages to different extents.

- Develop **interest** in learning mathematics.
- Show **keenness** to participate in mathematical activities.
- Develop **sensitivity** towards the importance of mathematics in daily life.
- Show **confidence** in applying mathematical knowledge in daily life, clarifying one's argument and challenging others' statements.
- Share ideas and experience and work **co-operatively** with others in accomplishing mathematical tasks/activities and solving mathematical problems.
- Understand and take up one's **responsibilities** in group work.
- Be **open-minded** in doing group work, willing to listen to others in the discussion of mathematical problems and respect others' opinions, value and appreciate others' contributions in doing mathematics together.
- **Think independently** in solving mathematical problems.
- Be **persistent** in solving mathematical problems.
- **Appreciate** the precise, aesthetic and cultural aspects of mathematics and the role of mathematics in human affairs.

Appendix 6

Exemplars

Exemplar 1 : Bead Game

Learning Dimension: Number

Learning Unit: Numbers to 10

Key Stage: 1

Objective: To develop an understanding of the composition of numbers 1 – 10.

Prerequisite Knowledge:

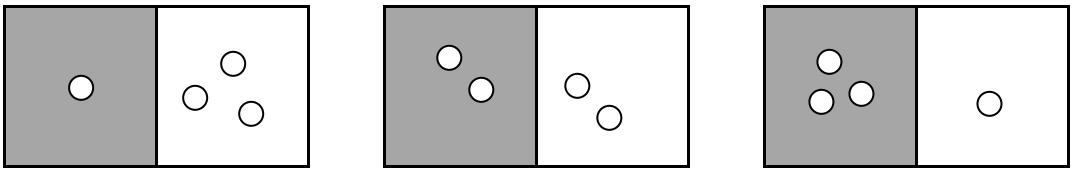
Pupils should be able to
(i) count and read the number of objects.
(ii) recognize odd and even numbers.
(iii) compare two types of objects by one-to-one correspondence.

Teaching Resources:
(i) Beads
(ii) Stickers
(iii) Worksheets (Bead Game)

Description of the Activity:

Activity 1:

1. Pupils are grouped in pairs. Each group of pupils is given with 10 beads.
2. Teacher asks pupils to take out 4 beads and group them into two sets (each set has at least 1 bead). Pupils are requested to explore the different ways of grouping that can be found and record their findings by using stickers on the worksheets as shown in the following figures.



Questions for Discussion:

1. How many different ways can be found by grouping 4 beads into 2 sets?
2. If one more bead is put in the shaded region, what is the change in the number of beads in the unshaded region?
3. If more and more beads are put in the shaded region, what is the change in the number of beads in the unshaded region?
4. How can you ensure that all combinations are found?

Notes for Teachers:

1. As pupils do not understand the concept of "0" at this stage, teachers should give hints to pupils to put at least 1 bead in the shaded or unshaded region.
2. If stickers are not used, teachers can ask pupils to draw their results on the worksheets.

Activity 2:

1. Pupils repeat the procedures in Activity 1 for grouping 2 to 10 beads into two sets and find the different compositions of each number.
2. Pupils record the results by using stickers on the worksheets.

Questions for Discussion:

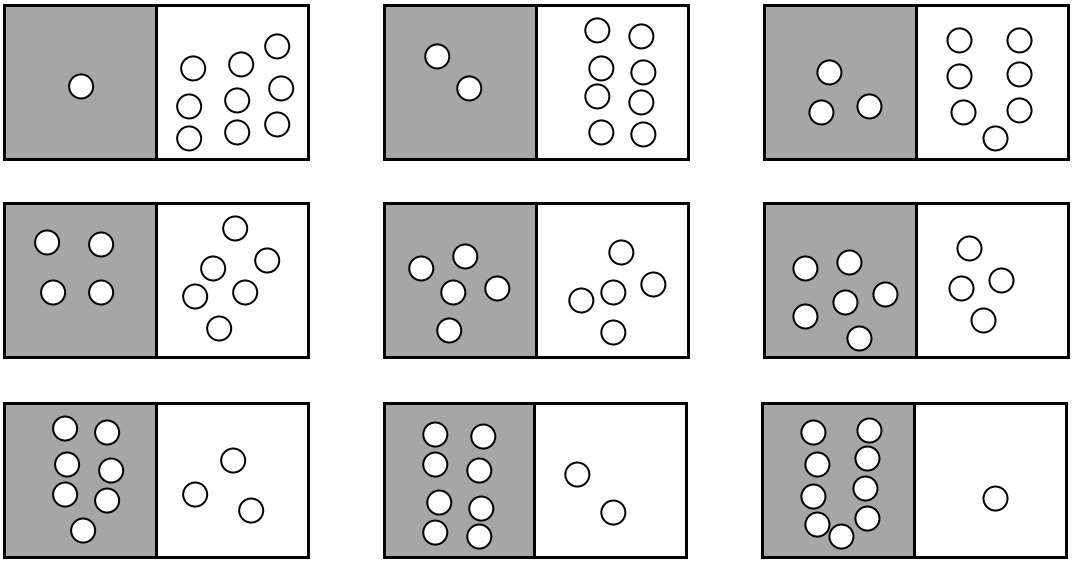
1. Repeat the discussion in Activity 1.
2. According to the results in Activity 2, which numbers have the same quantity of beads in the shaded and unshaded regions when grouping the beads into 2 sets? (2, 4, 6, 8, 10)
3. What is the characteristic of these numbers? (Teacher leads pupils to find out that these are even numbers.)

Notes for Teachers:

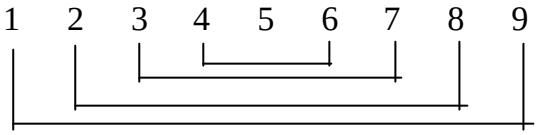
As pupils do not understand the concept of "0" at this stage, 2 to 10 beads should be used in the Activity.

Activity 3:

1. Teacher asks pupils to observe the results in Activity 2 by grouping 10 beads into two sets as shown in the following figures.



2. Teacher writes the numbers from 1 to 9 on the blackboard. Pupils then join the numbers together from the above results and describe the rule they find as follows.



Questions for Discussion:

1. Which number cannot be joined to another number? Why?
2. If we need to join a line to this number, what is its partner? Why?

This exemplar involves the following generic skills：

1. Critical Thinking
   - Understand straightforward cause and effect relationships, such as, adding one bead in the shaded region will cause decreasing one bead in the unshaded region.
   - Reason inductively in exploring ways of finding out all the combinations.
   - Reason inductively such as exploring the characteristic of numbers.
   - Draw logical conclusions based on adequate data and evidence, for example, by joining the number together by observing the results of grouping 10 beads into 2 sets.

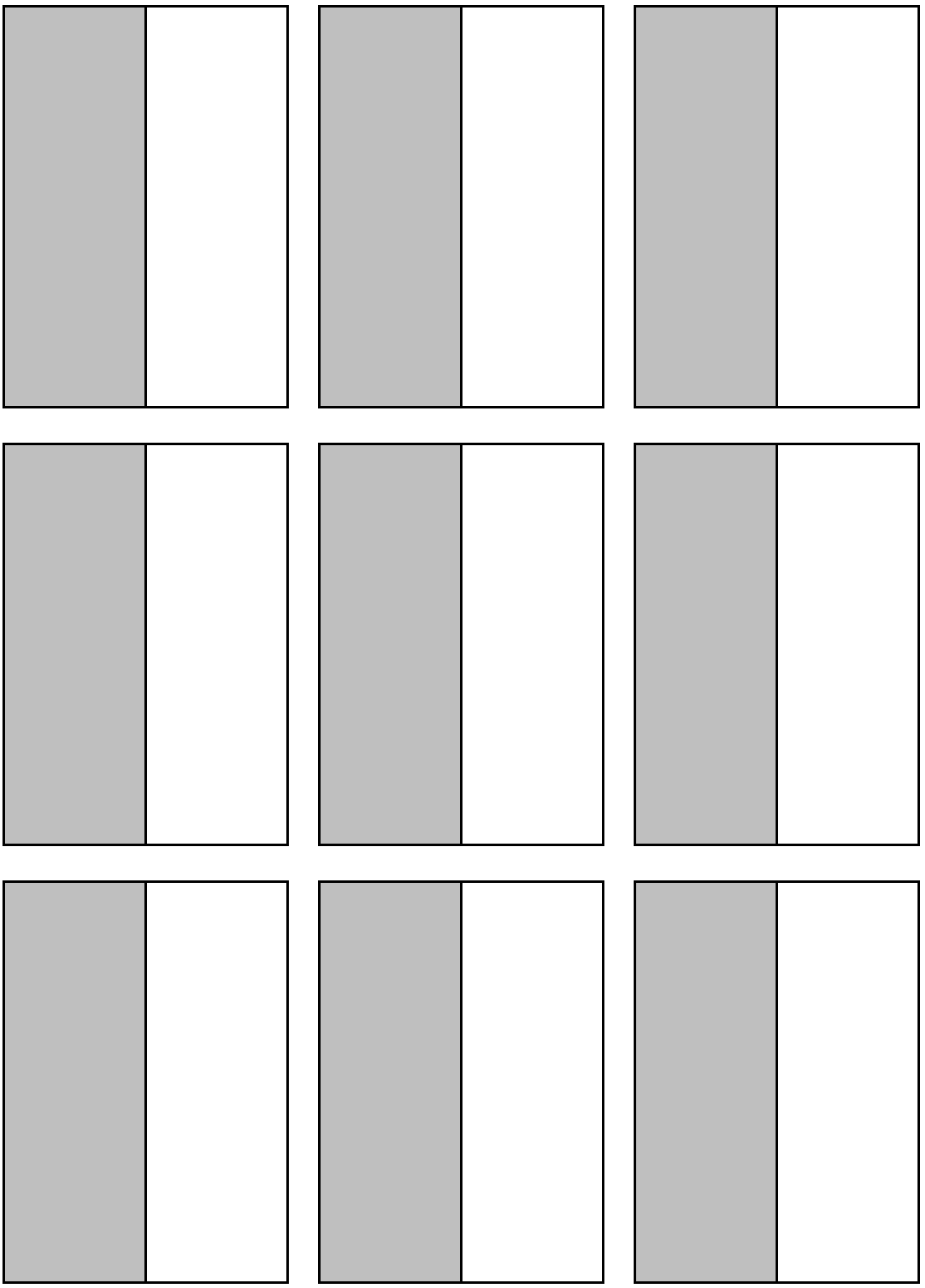
2. Communication
    - Comprehend and act appropriately on spoken instructions, for example, by following teachers' instructions to group the beads into 2 sets and to record the findings on the worksheet.
    - Present results of tasks with drawings and symbols.
    - Discuss with others in accomplishing tasks, for example, by grouping the beads into 2 sets.

3. Problem Solving
    - Learn from past experience to solve new problems in grouping 2 to 10 beads.
    - Adopt various ways of solving problems, for example, by finding out all combinations by transferring beads one by one from one region to another region or by putting some beads in one region and then the rest in another region.

Appendix 6

Bead Game

Group [ ] beads into two sets and record the results by putting stickers on the figures below.



Exemplar 2 ：Making Solids

Learning Dimension: Shape and Space

Learning Unit: 3-D shapes (III)

Key Stage: 2

Objectives: (i) To make solids with cubes.
(ii) To explore the methods used to make all possible solids with cubes.
(iii) To describe the solids made.
(iv) To group the solids made.
(iv) To describe the methods used in grouping the solids.

Prerequisite Knowledge:
Pupils should be able to identify and describe 2-D and 3-D shapes.

Teaching Resources: (i) Cubes
(ii) Grid paper

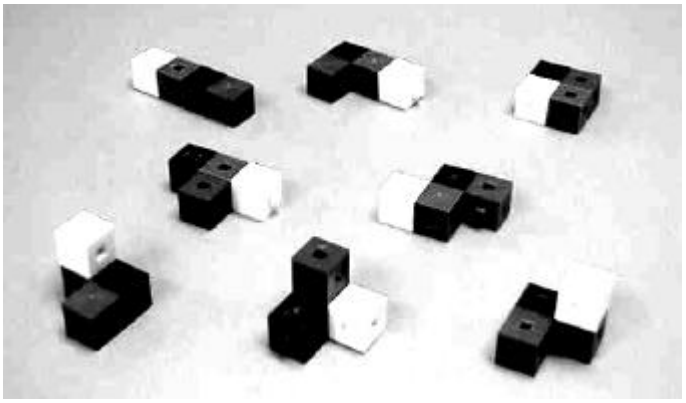
Description of the Activity:

Activity 1:
1. Teacher gives each pupil a box of cubes.
2. Pupils make solids with two cubes and display them on the desk.
3. Repeat the activity in (2) with three, four and five cubes.
4. Pupils are grouped in pairs to describe the solids to each other.
5. Pupils group the solids into several categories according to the characteristics of the solids.

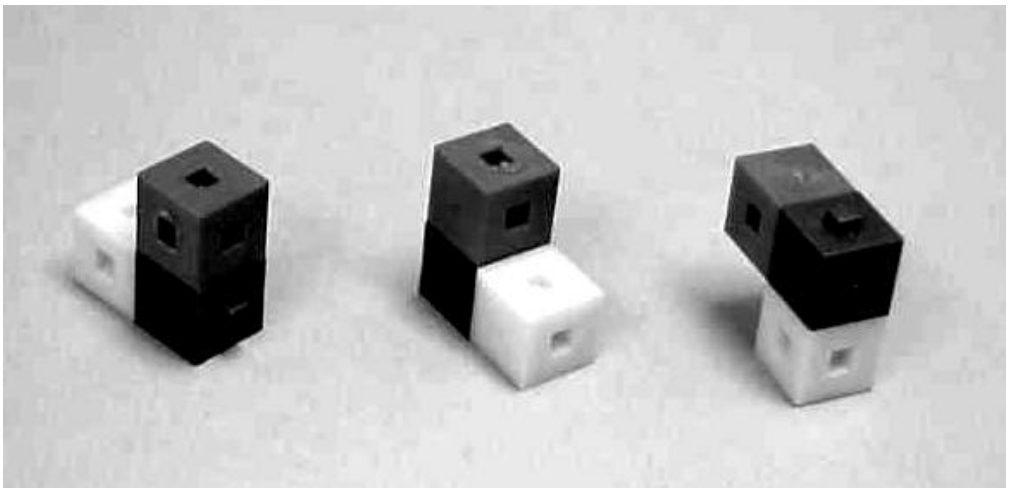
Questions for Discussion:
1. How many solids can be formed with two, three, four and five cubes?
2. Are you able to make all the possible solids? How?
3. What are the rules you followed in grouping the solids?

Notes for Teachers:

1. Teachers should let pupils explain freely their ways of making and grouping solids with cubes freely.
2. All the possible solids made with four cubes are as follows:

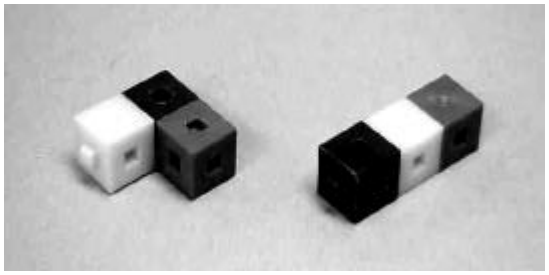


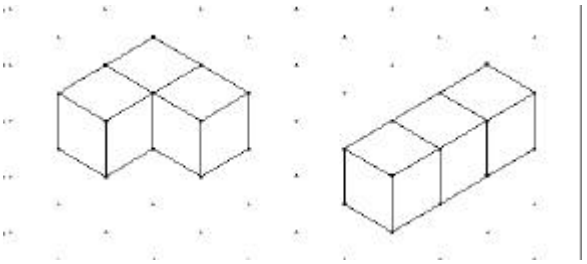
3. Any shapes which are formed by rotating or turning over other shapes are counted as identical.
   For example: the three solids below are considered to be the same.



4. Through observing pupils' performance in class, teachers can identify their strengths and areas for improvement in learning shapes. Ways to improve their spatial sense can then be sought.

Activity 2: (For more able pupils)
Record the solids made in Activity 1 on the grid paper.
For example: The following shapes are formed by using three cubes.





Notes for Teachers:
Teachers can prepare grid papers using suitable software. If Cabri Geometry is used, press "Define Grid" to show the axes. Move the x and y axes to form a grid.

This exemplar involves the following generic skills:

1. Communication
   - Describe the solids orally with simple and appropriate mathematical terms.
   - Explain orally the rules they followed in grouping the solids.
   - Present the results of tasks with appropriate drawings and symbols, for example, by recording the solids made in Activity 1 on the grid paper.

2. Critical Thinking
   - Categorize information using various criteria, for example, by grouping the solids into several categories according to the characteristics of the solids.
   - Reason inductively in the process of exploring ways of making all the possible solids.

3. Problem Solving
   - Use simple methods in solving problems, for example, by devising an appropriate plan for making solids.
   - Apply knowledge learnt from past experience to solve new problems, for example, by making solids with 4 and 5 cubes, based on the experience of making solids with 3 cubes.

Exemplar 3 ：Knowing Your Community (Cross-curricular project)

Learning Dimension: Shape & Space, Data Handling

Learning Unit: The eight compass points and bar charts (I)

Key Stage: 2

Objectives: (i) To find the eight compass points with a compass.
(ii) To become familiar with the district in the vicinity of the school.
(iii) To sketch the city plan of the district in the vicinity of the school.
(iv) To be aware of the facilities and services offered in the community.
(v) To present the collected data with statistical charts.

Prerequisite Knowledge:
Pupils should be able to: (i) find the four directions with a compass;
(ii) construct a simple bar chart.

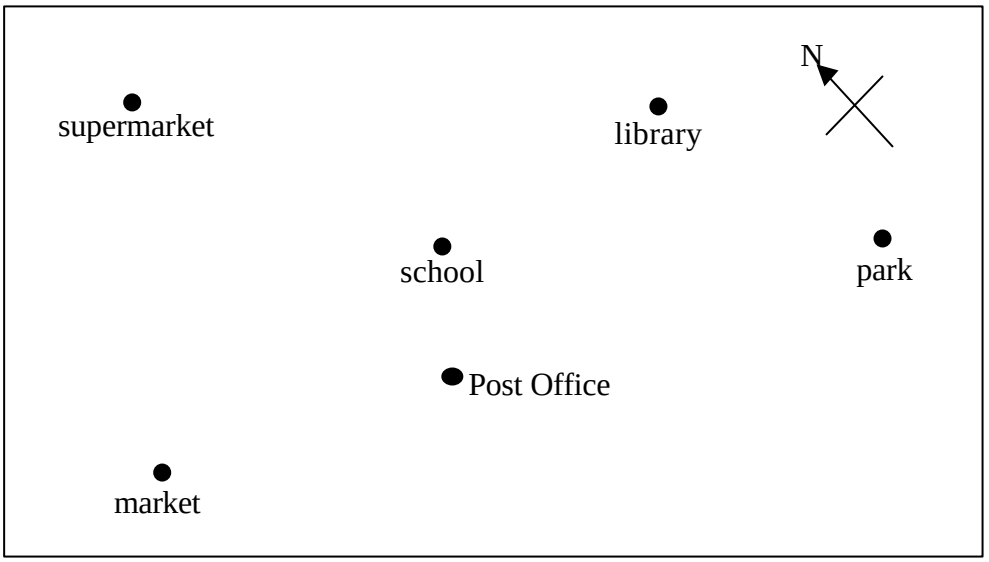
Teaching Resources: Compasses

Description of the Activity:

Activity 1:
1. Pupils use compasses to find the four directions and describe the facilities within the school vicinity in those directions.
2. Pupils use compasses to find the eight compass points and record the facilities within the school vicinity in those directions.

Questions for Discussion:
1. Will the directions of the facilities change when you are in different positions?
2. Will the directions of the facilities change when you are in the same position but face a different direction?

Activity 2:
Pupils are divided into groups to do the following tasks:
1. Sketch the city plan of the area in the vicinity of the school, with the help of compasses. For example,



(Since distances between facilities are not the main concern of this activity, pupils can estimate them intuitively.)

2. Collect information on the facilities and services available within the school vicinity, for example, the number of supermarkets, parks, markets, libraries, the condition and maintenance of the facilities, the frequency with which the facilities are used by the residents, etc.
   (More able pupils can collect information by conducting a small-scale survey.)
3. Analyze the information collected and present the data with appropriate statistical charts, such as bar charts. Teachers can encourage pupils to make use of new technology in presenting their findings and reports. For example, pupils can record data and draw statistical charts by spreadsheet, compile the report by word processor and take pictures of the facilities with digital cameras, etc.
4. Discuss the statistical charts constructed and report the findings orally or in written form.
5. Suggest ways to improve the city planning of the area in the vicinity of the school.

Questions for Discussion:

1. What kinds of facilities are provided in the district?
2. Which types of facilities do you think are in excess? Why?
3. Which types of facilities do you think are insufficient? Why?
4. If you were a town planner, how would you redesign this district?
5. How can you protect and improve the environment of the community?

Notes for Teachers:

1. Pupils should be given adequate time to do the project.
2. Teachers should give explicit instructions and suggest ways to collect data.

This exemplar involves the following generic skills:

1. Collaboration
   - Share responsibilities and understand the roles of individual members in collecting data and presenting projects.
   - Discuss and exchange ideas openly with others in sketching the city plan and deciding information which needs to be collected.
   - Listen to others patiently in discussions of the information collected.
   - Understand and accept different interpretations in analyzing statistical data and ways of improving the city plan from members with different socio-economic backgrounds
   - Participate actively in the project, for example, in discussing strategies to be adopted for collecting information on the facilities.
   - Agree a suitable strategy to collect information.
   - Liaise, negotiate and compromise with others in selecting a suitable strategy for collecting information and in suggesting a new city plan.

2. Critical Thinking
   - Extract, classify and organize information collected from the area in the vicinity of the school.
   - Understand the concepts of relevance and irrelevance, for example, in identifying the relevant information useful for supporting the proposed city plan.
   - Draw logical conclusions based on adequate data and evidence, for example, in making suggestions for improving the city planning, based on the information collected.

3. Problem Solving
   - Choose the relevant information to solve problems, for example, by identifying the statistical data necessary for giving suggestions for improving the city planning.
   - Make use of various tools in solving problems, for example, using a tally in counting the frequency with which the facilities are used and ballots to collect opinions about the level of

satisfaction with the condition of the facilities.

4. Creativity
   - Strengthen creative abilities such as originality, sensitivity to problems, problem formulating and analysis, through the process of designing their own projects, presenting the city plan in their own ways and suggesting ways of improving the city planning.

5. Numeracy
   - Recognize the 8 compass points.
   - Organize simple data and interpret simple statistical graphs in daily-life situations, for example, in investigating the facilities within the school vicinity.
   - Choose the correct form of numbers in presenting information, for example, by using percentages to tell how many people are satisfied with the conditions of the facilities.

Exemplar 4: Graphical Representation of Linear Equations

Learning Dimension: Number & Algebra

Learning Unit: Linear Equations in Two Unknowns

Key Stage: 3

Objective: To explore the graphical representation of linear equations in 2 unknowns with the help of IT tools

Prerequisite Knowledge: (i) Using paper and pencil to plot graphs of simple linear equations in 2 unknowns.

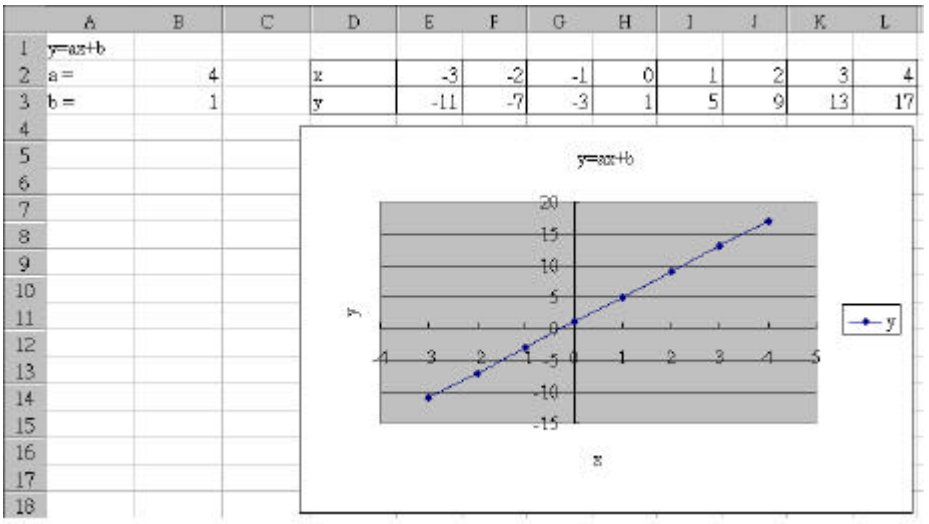
(ii) Basic manipulation skills on the hardware and spreadsheet software or graphing calculators.

Teaching Resources: Spreadsheet software such as EXCEL or graphing calculators

Description of the Activity:

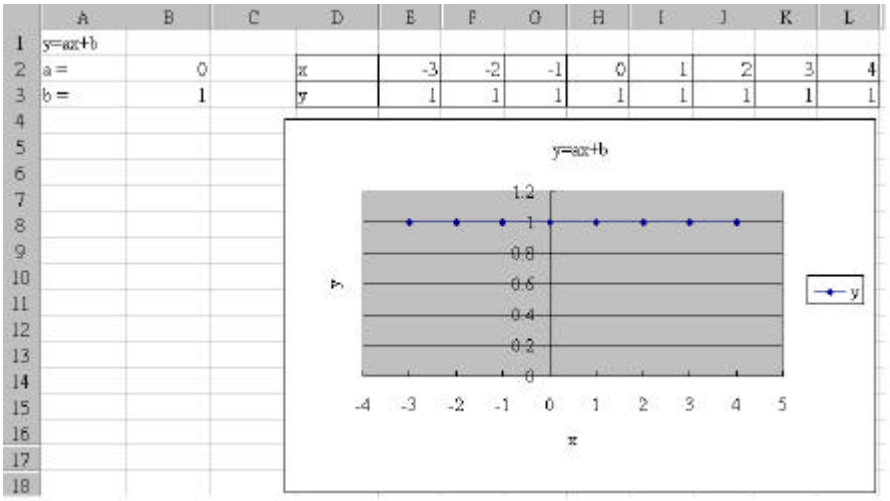
1. After asking students to plot the graph of the simple linear equations with paper and pencil, the teacher can then ask students to explore graphs of different linear equations with the software EXCEL or with graphing calculators.
2. The following worksheet is prepared. A brief explanation of the set-up of the worksheet is given. It includes:
   - the set-up of cells A1, A2, A3 for the text of the equation name and the values for a and b;
   - values of x to be keyed in or pre-set in the worksheet;
   - values of a and b to be keyed in;
   - the definition of y from the corresponding values of x.

   Besides the algebraic language, the teacher may introduce the definition of y in EXCEL language.



3. Students are asked to explore graphs of different equations in the following categories:
   - Set b=0 and explore the shape of the graphs of y = ax by changing values of a.
   - Set a=1 and explore the shape of the graphs of y = x+b by changing values of b.
   - Set b to any constant values and explore the shape of the graphs of y = ax+b by changing values of a.

   Students are asked to contrast the steepness and the intercepts of the resulting graphs. Discussions on the graphs of these categories of equations can then be held.

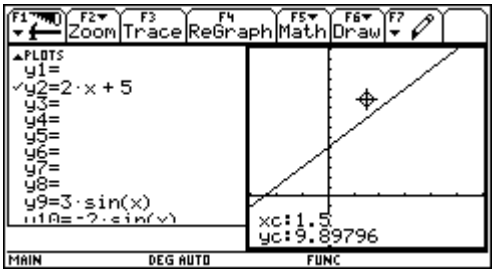


Notes for Teachers:

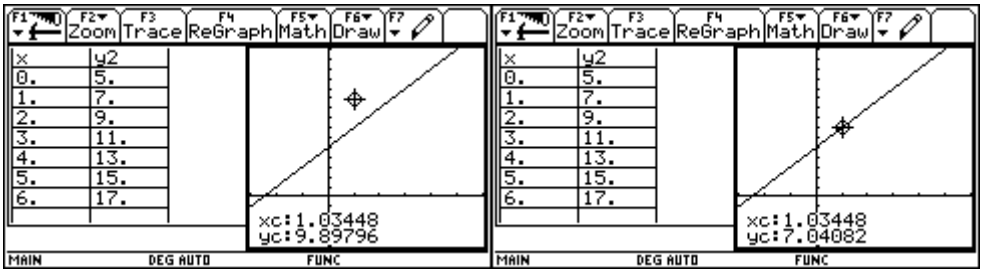
1. Teachers can prepare the worksheet prior to the lesson so that students can spend more time in exploring graphs of different equations. The steps in preparing the worksheet are as follows:
   - Define A1 as “y = ax + b”.
   - Define A2 as “a=”, A3 as “b=”, D2 as ”x” and D3 as “y”.
   - Key in values of cells B2 and B3 as 0 and 1 respectively.

- Key in –3, -2 to the cells E2 and F2 and then copy these to other cells in the row.
- Define the values of y in the cell E3 as "=$B$2*E$2+$B$3".
- Copy this definition to other cells of y. The values of y will be generated automatically by the software.
- Highlight values of x and y and then use the SCATTER LINE GRAPH graphing facility to draw the graph.

2. Other graphing software such as WinPlot.exe, which can be downloaded from the Internet, can be used to serve a similar purpose. Keying the equation into the software can draw the graph. Alternatively, some graphing calculators also provide such facilities. EXCEL is used for this demonstration because of its availability in most schools. Its facility in showing tables and graphs of different equations on one screen is an advantage to facilitate students to visualize the change of equations on the numerical and graphical values simultaneously. Below are some examples using graphing calculators in the exploratory activity. Students can use the TRACE function built into the graphing calculators to explore points on the line and the relation of the coordinates of the points with reference to the equation.



Students then observe the coordinates of points not lying on the straight line and compare the coordinates with those on the straight lines.



Teachers should remind students of the problems they will encounter when tracing the coordinates of the points on the straight line. (e.g. It is not easy to "trace" a point on the straight line and y does not equal ax+b exactly, due to rounding error.)

3. Students familiar with the EXCEL platform can set up their own worksheet. They can design their own styles of graphical presentation and tabular representations. The program is then no longer a black box to students. The definition of y also enhances students' understanding of the concepts of variables and constants.
4. The program can be modified to have the tables and graphs of 2 equations (or more) presented simultaneously on one screen. This helps students to compare the graphs of different equations.
5. After this activity, students are expected to recognize that
   - the shape of a linear equation in 2 unknowns is a straight line;
   - the line with the equation y = ax passes through the origin and the inclination of the line depends on the values of a;
   - the equation of y = b is a straight line parallel to the x-axis;
   - the value of b is the y-intercept of the equation y = ax + b.

This exemplar involves the following generic skills:

1. Information Technology
   - Use computer software EXCEL or graphing calculators to carry out the exploratory activities.

2. Numeracy
   - Use different values of a and b for y=ax+b to investigate the effects of changes in these values on the shape of the graph.
   - Observe the graph and make estimates of the values of b.

3. Critical Thinking
   - Compare the values used for a and b in y=ax+b and note the effects of the changes in these numbers on the shape of the graph and hence draw conclusions about their findings in discussion.

4. Problem Solving
   - Explore the first part of the problem (i.e. by setting b=0) by following the procedures suggested in the exemplar.
   - Adjust their strategy of exploration, for example, using ascending numbers or descending numbers.
   - Plan and adopt an appropriate strategy by designing their own styles of graphical presentation and tabular representation.

Remark: At present, teachers quite often use graph board or overhead transparency to illustrate the graphs of linear equations in two unknowns. However, they are not effective methods as only a few equations could be drawn in a lesson and it is not flexible enough for students to do exploration. The following merits can be obtained with the use of IT:

- A great variety of graphs could be shown within seconds.
- It allows students to propose their own equations and make guess on the shapes of the graphs.
- With the numeric (tabular), graphic, and algebraic representations of the equations shown on one screen, students can see the changes in the numeric and graphic forms of the equations simultaneously whenever there is change in the algebraic form. This presentation facility can help students build up a stronger sense of the meaning of algebraic forms of equations.

Exemplar 5: Graphs of Quadratic Functions, $y=ax^2+c$

Learning Dimension: Number & Algebra

Learning Unit: Functions and Graphs

Key Stage: 4

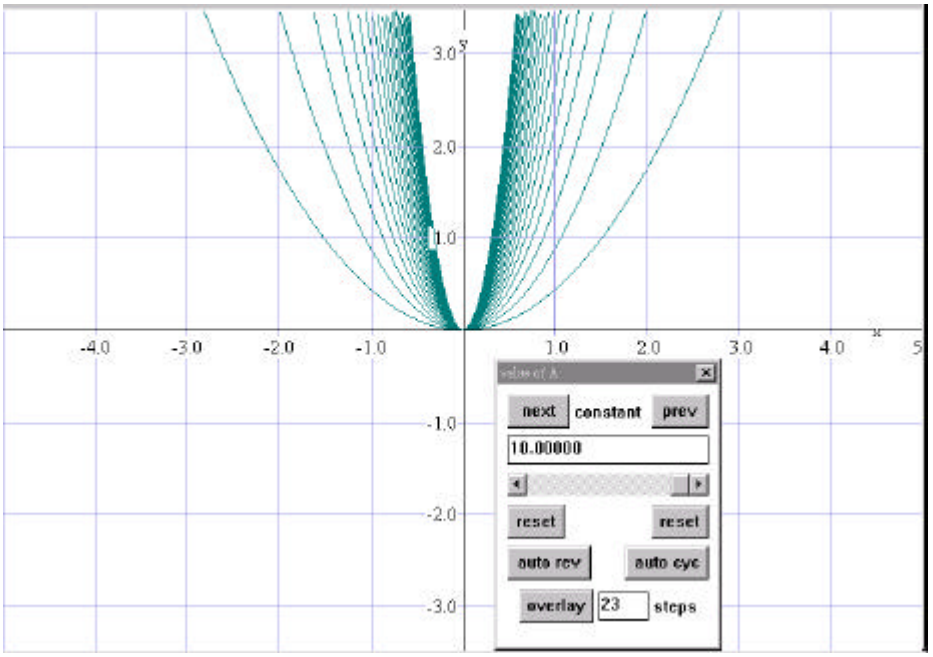
Objectives: To investigate the graphical meaning of a and c in the quadratic function $y=ax^2+c$ with the help of IT tools

Prerequisite Knowledge:
(i) The notation and basic concepts of functions
(ii) Intuitive ideas on the shape of the quadratic function $y=ax^2+bx+c$
(iii) The shape of the graph $y = x^2$

Teaching Resources: Spreadsheet software or WinPlot.exe downloaded from the web site:
http://academy.exter.edu/~rparris

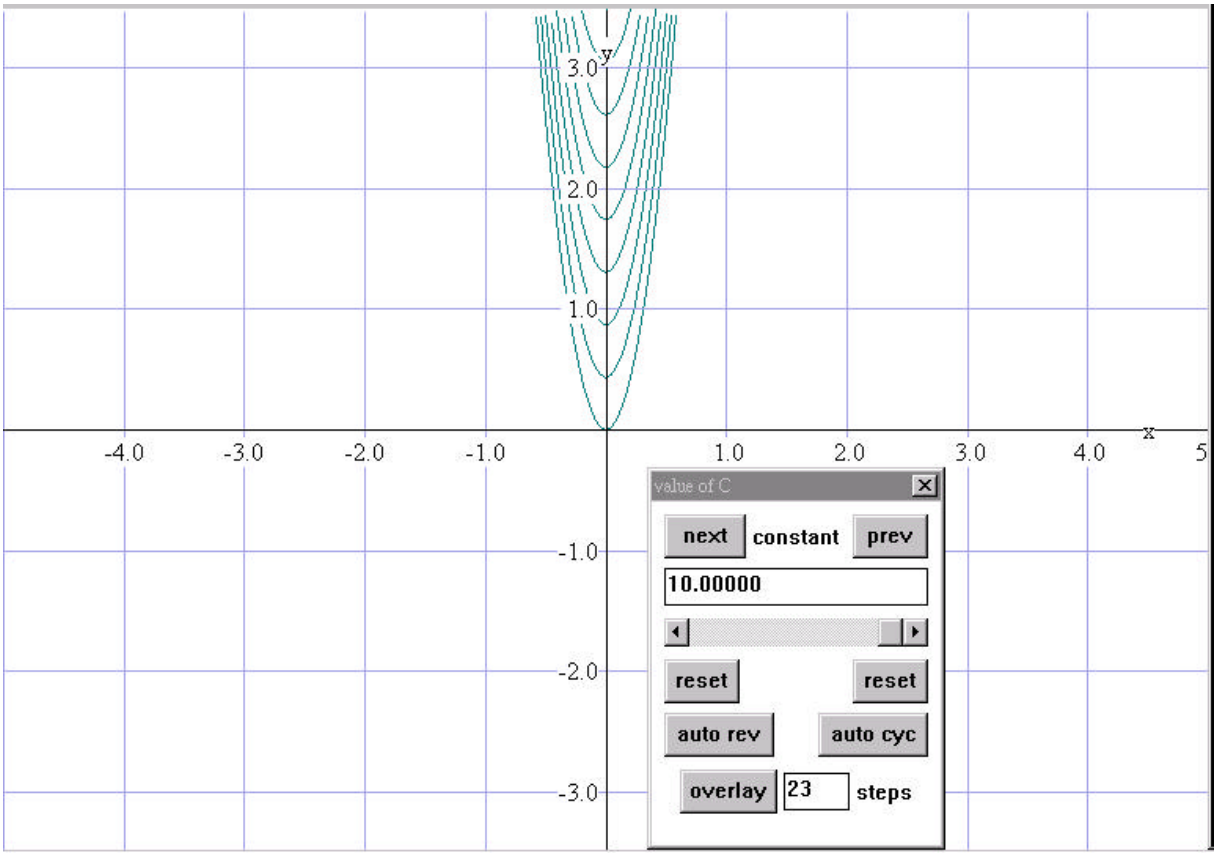
Description of the Activity:

1. Students are divided into groups. Then they are asked to work on the WinPlot platform to explore the quadratic function $y = ax^2 + c$
2. The value of c is set to zero. Students are asked to explore the shape of the function with the value of a changing from 1 to 10 and also from –1 to –10. Students are asked to observe the changes of the shapes of the graphs with increasing values of a. The animation can be produced by the software. Similarly,



observations and discussions should be made when the value of a becomes negative.

3. By setting the value of a to 1, students are asked to observe the changes of the graph for different values of c.



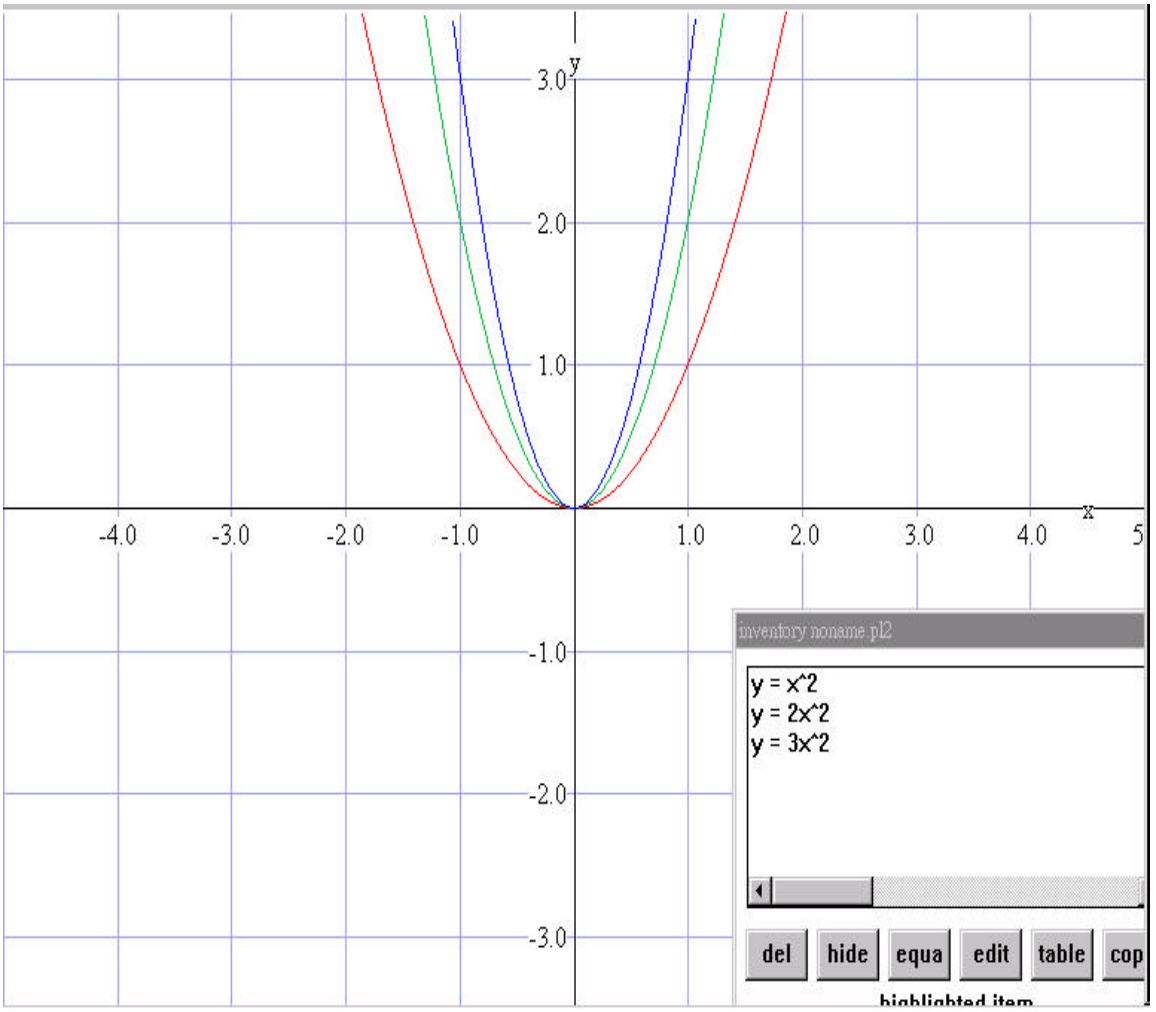
4. Students are then asked to make conjectures on the shapes of the following graphs. They are given the graph of $y = x^2$. They should sketch their conjectures on the diagram provided.
   a) $y = 2x^2$
   b) $y = 2x^2+3$
   c) $y = 4x^2-5$
   d) $y = -3x^2+4$

   Discussions on their conjectures should be made. The graphical meaning of a and c of
   $y = ax^2 + c$ should be discussed and summarized.

Notes for Teachers:

1. The teacher should remind students that the notation of x^2 in the program is x^2. This notation is commonly used in many software packages or computer languages.
2. The diagrams from WinPlot in sketching the quadratic function can be adjusted. If the teacher finds that students cannot follow the animation produced, the program can be adjusted to have the

diagrams drawn one by one.



3. At the end of the activity, students are expected to recognize that
   - the graph is symmetrical about the y-axis
   - the coefficient of x^2, a, affects the direction of opening and the slope of the curve
   - the constant c is the y-intercept of the parabola
   - the constant c in $y = ax^2 + c$ is the translation of $y = ax^2$ along the y-axis.
   - the minimum or the maximum value of the function is c.

This exemplar involves the following generic skills:

1. Information Technology
   - Make use of the computer software *Winplot* to do their own investigations.
   - Use the animation function provided by the software to

explore the effect of coefficients on the quadratic graphs.

2. Numeracy
   - Understand the meaning of the mathematical formula $y = ax^2 + c$.
   - Investigate the effects on the graphs of the quadratic function by using different values of a and b.

3. Critical Thinking
   - Discover the properties of quadratic functions, classify them according to their properties (e.g. opening upwards / downwards, narrow / flat when compared with the graph $y = x^2$; shifting the vertex and etc) and generalize their observations from several examples.
   - Predict the graphs of other functions and make use of the software to check their predictions.

Remark: Teachers quite often use graph board or overhead transparency to illustrate graphs of quadratic functions. Clearly, they are not effective methods as only a few graphs could be drawn in a lesson and it is not flexible enough for students to explore other equations not prepared. The following merits can be obtained with the use of IT:
   - A great variety of graphs could be drawn and shown within seconds.
   - It allows students to explore different functions and check their conjecture on the relation between the shapes of the graphs and their functions.
   - With the facility of showing several graphs on one screen, it is easier for students to compare graphs of functions with different coefficients, different constant terms, etc.

Exemplar 6: Bus Reports

Learning Dimension: Data Handling

Learning Unit: Uses and Abuses of Statistics

Key Stage: 4

Objectives: (i) To enhance students' interest and understanding in analyzing and comparing data
(ii) To develop students' critical thinking skills in examining the interpretations of the set of given data.

Prerequisite Knowledge: Fundamental knowledge and simple manipulations of percentages

Teaching Resources: Information regarding the publicity and advertisement for bus routes, bus operations etc.

Description of the Activity:

1. The bus is one of the main means of mass transportation in Hong Kong and is familiar to students. With reference to the exploration of the routine operations of bus companies, students are expected to strengthen their knowledge and to care about things around them. Students will understand the importance of background information in the interpretation of data analysis.
2. Teachers can raise the following problems for motivation:
   a) How many public bus companies do we have in Hong Kong?
   b) Compare the three cross-harbour tunnel routes connecting Hong Kong Island and Kowloon Peninsula with regard to the following:
      (i) the cross-harbour tunnel bus routes;
      (ii) the lengths of the cross-harbour tunnels;
      (iii) the location of the entrance to each tunnel;
      (iv) the toll for each tunnel;
      (v) the traffic congestion for each tunnel.
   c) Will an increase in the frequency of buses crossing the tunnels worsen the traffic congestion inside the tunnels?
3. After giving the following table to students, ask them to investigate the problems concerning the punctuality of bus route schedules for two different bus companies.

Table 1 The punctuality of bus route schedules for two different bus companies

Route	Company A		Company B	
	Punctuality (%)	Frequency	Punctuality (%)	Frequency
I. Via Cross-harbour Tunnel to Sai Ying Pun	86.7	922	78.5	534
II. Via Cross-harbour Tunnel to North Point	82.5	2056	74.1	312
III. Via Cross-harbour Tunnel to Chai Wan	94.1	568	92.2	1534
IV. Via Western Harbour Tunnel to Tsuen Wan	96.2	265	93.7	1653
Total	86.2	3811	89.6	4033

Problems for Discussion:

1. How do you calculate the overall punctuality percentage for each company in Table 1?
2. If the punctuality for each bus route is compared solely from the information provided in the above table, it is obvious that the performance of company A is much better than that of company B. However, why is the overall performance of the former worse than that of the latter?
3. If it is assumed that the delays of the bus schedules are due to traffic congestion, as claimed by the drivers of bus routes I and II, is this point valuable in comparing the punctuality of the schedules of the two companies?

Notes for Teachers:

- For problems raised in question 2, teachers can arrange a short Question-and-Answer competition for students, in order to identify their understanding of the background information.
- For question 1, the method of calculation for "overall

punctuality percentage" in Table 1 is as follows.

- The overall punctuality percentage of bus company A

$$= \frac{86.7 \times 922 + 82.5 \times 2056 + 94.1 \times 568 + 96.2 \times 265}{922 + 2056 + 568 + 265} = 86.2$$

[The percentage for that of bus company B can be found in a similar way.]

- For questions 2 and 3, teachers should guide students to observe the following from Table 1.
  - For bus company A, the "Punctuality" percentage is the **lowest** for the route with the highest frequency. On the other hand, the "Punctuality" percentage is the **highest** for the route with the highest frequency for bus company B.
  - The background information provided by the drivers indicates that bus company A made a mistake by putting a large number of bus routes into the congested areas, leading to the lowering of the overall performance.

This exemplar involves the following generic skills:

1. Communication
   - Use mathematical language to explain how to find the overall punctuality percentage for each company.
   - Make their own reasonable arguments from the relevant evidence.

2. Numeracy
   - Apply the correct formula to calculate the overall punctuality percentage from individual percentages.

3. Critical Thinking
   - Judge and evaluate the contradicting arguments arising from Problem 2.
   - Make their own arguments that greater individual punctuality percentages may not give rise to a greater overall punctuality percentage.