

**2018/19 The 10th Hong Kong Mathematics
Creative Problem Solving Competition
for Secondary School (Heat)**

School-ID:	
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Time allowed : 50 minutes

Instructions for participants :

1. Time allowed: 50 minutes.
2. The question paper consists of 13 pages. There are 15 questions in this paper.
3. Each participant will get one set of question paper.
4. Each team will get **a set of answer sheets and two pieces of square paper (Appendix A)**.
5. Team members are allowed to discuss during the competition. The agreed answers should be written on the answer sheets.
**** Only the answers in the answer sheet will be marked.**
6. Participating teams should bring their own stationery and calculators. For the purpose of fairness, please use only non-graphic calculators. Electronic dictionaries, computers, mobile phones and other communication devices are prohibited.
7. The blank space on each page of this question paper can be used for rough work. Each participant will get one rough work sheet. Extra rough work sheets will be provided upon request.
8. The question papers, answer sheets, square papers and rough work sheets will be collected after the competition. Participants are not allowed to take away any of these papers or the team might risk disqualification.

2018/19 第十屆香港中學數學創意解難比賽(初賽)

學校編號:	
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比賽時間：50 分鐘

參賽者須知：

1. 比賽時間：50 分鐘。
 2. 本問題卷共 13 頁，全卷共有 15 題。
 3. 每位參賽學生獲派一份問題卷。
 4. 每隊參賽隊伍獲派一份答題紙及兩張正方形紙 (附件 A)。
 5. 比賽其間隊員可以討論題目，並於答題紙寫上議定的答案。
** 只有寫於答題紙上的答案方可得到評分。
 6. 參賽隊伍需自備文具及計算機。為公平起見，比賽中只可使用非圖像計算機。本比賽中嚴禁使用電子字典、電腦、電話或其他有上網或通訊功能的工具。
 7. 本試卷每頁的空白位置可作為草稿之用。每位參賽學生會獲派一張草稿紙，如有需要，可要求額外草稿紙。
 8. 在筆試完結後，各學生必須交回所有問題卷、答題紙、正方形紙及草稿紙。參賽學生不得取走任何於比賽中所派發之紙張文具，違規者全隊可被取消資格。
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1.

已知 $23! > 2019^n$ ，求 n 的最大可能整數值。

當中 $23!$ 為 23 之階乘，是 1 至 23 (包括 1 和 23) 內所有自然數的乘積。

例: $5! = 5 \times 4 \times 3 \times 2 \times 1$

Given that $23! > 2019^n$, find the largest possible integer n .

$23!$ denotes 23 factorial, the product of all the natural numbers from 1 to 23 (inclusive).

For example, $5! = 5 \times 4 \times 3 \times 2 \times 1$

2.

A、B、C、D 和 E 點分別是 5 個圓的圓心。各圓的半徑是 3 厘米。如圖 1 所示，所有圓相交。

求整個圖形的周界(實線部分)。答案以 π 表示。

Points A, B, C, D and E are the centers of five circles respectively. The radius of each circle is 3 cm.

The circles intersect as shown in figure 1. Find the perimeter of the entire figure (**marked in solid line**). Give the answer in terms of π .

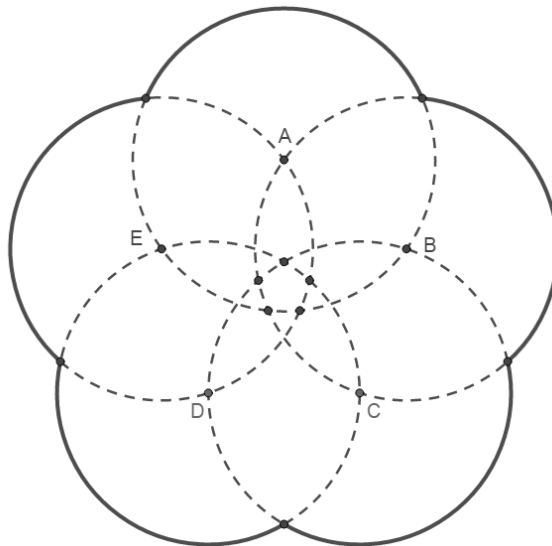


圖 1 / Figure 1

3.

在 $n \times n$ 的方格內，包含了數字格、地雷格 (×) 和安全格 (○)。數字格中的值顯示周邊有多少個地雷格 (包括垂直、水平或對角)。圖 2a 是一個例子。

In $n \times n$ grid, there are Number grids, Mine grids (×) and Safe grids (○). The number inside the Number grid indicates the number of Mine grids surrounding it (vertically, horizontally, or diagonally). Figure 2a shows an example.

○	○	1	○
○	3	×	2
×	○	×	2
1	2	1	○

圖 2a / Figure 2a

圖 2b 空白的位罝隱藏了地雷格和安全格，請把它們找出來，並分別以 (×) 和 (○) 表示。

In figure 2b, the Mine grids and Safe grids are hidden in the blank space. Find them out and label them by (×) and (○) respectively.

2			2	3	
	2				
			3		1
	4				
			1		
2					1

圖 2b / Figure 2b

4.





圖 3a 是一個由正四面體表面組成的立體迷宮，現把迷宮攤開成一摺紙圖樣（圖 3b）。請在圖 3b 上由起點  畫出路徑至終點 。（請清晰顯示路徑，並清除非路徑的線段。）

Figure 3a shows a 3D maze made by the surface of a regular tetrahedron. It is unfolded as a net (figure 3b). Draw the path from the starting point  to the end point  in figure 3b. (Show your path clearly and all the unwanted paths should be erased.)

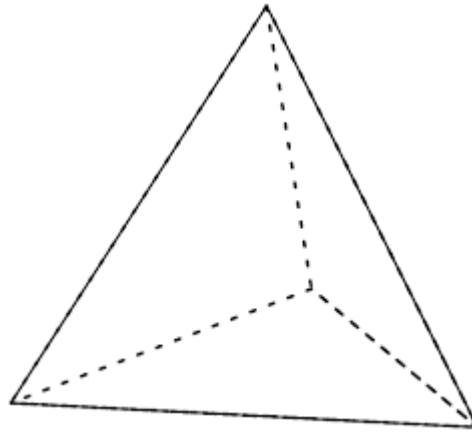


圖 3a / Figure 3a

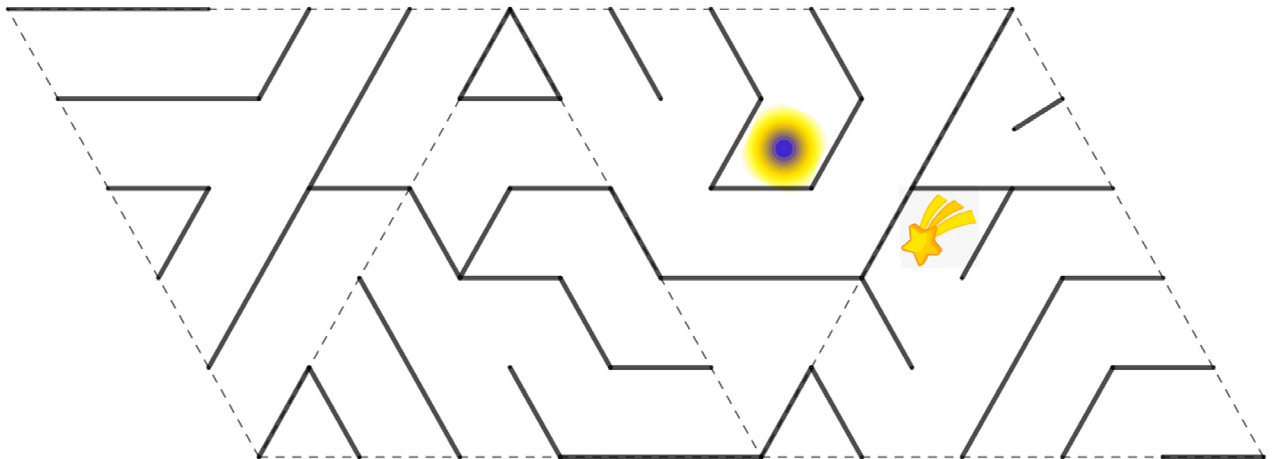


圖 3b / Figure 3b

5.

巴士路線 23 號由觀塘碼頭開往順天，服務時間為早上 06:10 至午夜 00:25，每 12 至 25 分鐘開出一班。其中一巴士準確地於某時某分正離開車站後，以每小時 44 公里的平均車速行駛了 12 公里。那時司機瞥見手錶上的分針和時針重疊。

Bus route number 23, from KWUN TONG FERRY to SHUN TIN, serves from morning 06:10 to midnight 00:25 in every 12 to 25 minutes. A bus left the terminal precisely on the minute. The average speed over the first 12 km was 44 km/h. Meanwhile, the driver consulted his watch and saw the hour-hand was overlapping with the minute-hand.

(a) 求巴士行駛該 12 公里路段所需的時間，以分鐘作單位，並以真分數表達。

Find the time needed for the bus travelled 12 km in minutes and express your answer in proper fraction.

(b) 請問巴士是何時離開車站？(以 24 小時報時制式表達)

At what time did the bus leave the terminal? (express your answer in 24-hour notation)

6.

已知 $x < y < z$ ，求下列方程組之解。

Given that $x < y < z$, solve the following system of equations.

$$\begin{cases} x^2 + y^2 + z^2 = 1341 \dots\dots\dots (1) \\ x + y + z = 63 \dots\dots\dots (2) \\ x - y = y - z \dots\dots\dots (3) \end{cases}$$

7.

400 至 800 間的整數 (包括 400 和 800)，共有多少個能符合以下**所有**條件：

- (I) 不能被 5 整除
- (II) 不能被 6 整除
- (III) 不包含數字「5」
- (IV) 不包含數字「6」

For all integers between 400 and 800 (inclusively), how many of them fulfill **ALL** the following conditions:

- (I) **NOT** divisible by 5
- (II) **NOT** divisible by 6
- (III) **DO NOT** contain digit “5”
- (IV) **DO NOT** contain digit “6”

8.

- (a) 圖 4a 為一正方形，頂點分別為 A, B, C 和 D。開始時，4 隻靜止的螞蟻分別位於各頂點上。一會兒後，每隻螞蟻都會隨機選擇一條邊，沿着該邊走到相鄰的頂點。問螞蟻之間有多少種不會相遇的走法？

Figure 4a shows a square with vertices A, B, C and D. In the beginning, 4 ants sit at different vertices. After a while, each ant moves to the adjacent vertex by randomly choosing and following a side. How many ways can the ants move such that they will not meet each other?

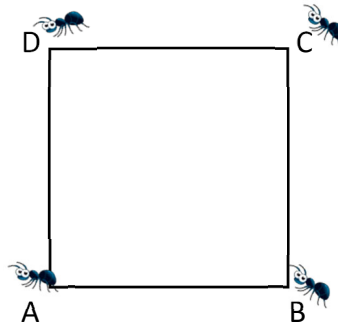


圖 4a / Figure 4a

- (b) 若(a)部的正方形改為正立方體 $ABCDEFGH$ (圖 4b) 及螞蟻數量由 4 隻改為 8 隻。螞蟻行走的方法與(a)部相同。問螞蟻之間有多少種不會相遇的走法？

Suppose the square in part (a) is changed into a cube $ABCDEFGH$ (figure 4b) and the number of ants is increased from 4 to 8. The movement of the ants follows the rules as part (a). How many ways can the ants move such that they will not meet each other?

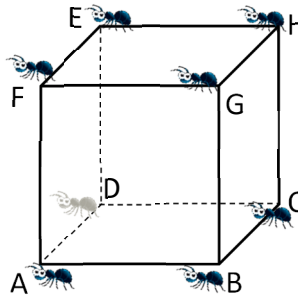


圖 4b / Figure 4b

9.

- (a) 求有多少種把九個相同的球分作三份的方法，使每一份至少有一個球，而每份中球的數量皆不同。

Find the number of ways to divide 9 identical balls into 3 groups, such that each group has at least 1 ball and the numbers of balls in each group are different.

- (b) 試利用下圖的等距方格來畫一個邊長分別為 1、2、3、4、5 和 6 的六邊形。

Using the isometric grid paper below, draw a hexagon with sides 1, 2, 3, 4, 5 and 6 respectively.



- (c) 求符合(b)部而面積最大的六邊形的面積。

Find the area of hexagon which has the largest area and satisfies the conditions in part (b).

10.

數名同學到食肆一起吃小籠包，當中一些「肚餓」的同學，他們每人會吃六隻或七隻小籠包，其他同學則每人吃一隻或兩隻。每一籠小籠包有六隻。若他們點三籠，則不能夠滿足所有同學；若他們點四籠，就會吃不完所有小籠包。

問共有多少同學到食肆吃小籠包？當中有多少是「肚餓」的同學？

Several students go to a restaurant to eat steamed pork buns in bamboo steamer. Each "hungry" student eats either 6 or 7 pork buns. Everyone else eats only 1 or 2 pork buns. Each bamboo steamer has 6 steamed pork buns. Three bamboo steamers are not sufficient to serve all students while students cannot finish all the pork buns if four bamboo steamers are ordered.

How many students went to the restaurant? How many of them were "hungry"?

11.

圖 5 顯示一個「希羅五邊形」，其邊長、對角線及面積均為自然數。

$AB = AE = 65$ ， $AC = AD = 156$ 及 $BCDE$ 是一個長方形。求 BD 的長度。

Figure 5 shows a Heron pentagon in which the sides, the diagonals and the area are natural numbers.

$AB = AE = 65$, $AC = AD = 156$ and $BCDE$ is a rectangle. Find the length of BD .

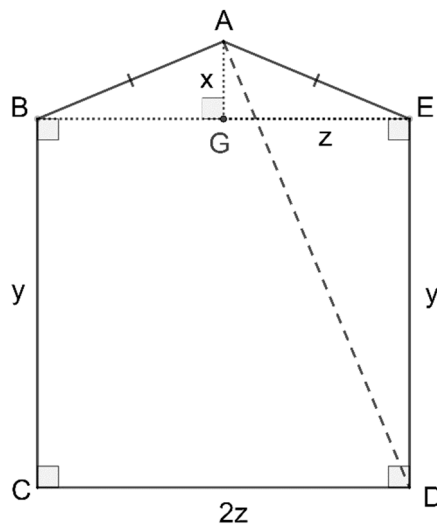


圖 5 / Figure 5

12.

有一位工程師正步行穿越一條隧道作結構檢查，在隧道中每兩逃生出口之間都有 4 個距離相等的標距指示牌作位置標示。當他行經其中兩逃生出口 G 和 H 時，控制中心通知他有一輛工程車向他迎面而來。在此刻他正位於由出口 G 去 H 之間的第 3 個標距指示牌，他判斷若向前跑，剛好趕及走進逃生出口 H；若向後跑，也剛好趕及走進逃生出口 G。若他每分鐘能跑 300 米，問工程車之速度每小時有多少公里？

An engineer is passing a tunnel on foot for structure inspection. Inside the tunnel, 4 distance marks are evenly distributed between every two safety exits. When he is walking on the way between safety exits G and H, control center informs him that a truck is coming toward him head-on. At that moment, he is at the third mark on the way from exit G to H. He realizes that he has just enough time to run toward the truck and get into the safety exit H or to run away from the truck and get into another safety exit G. If he can run 300 meters per minute, how fast is the truck going in km per hour?

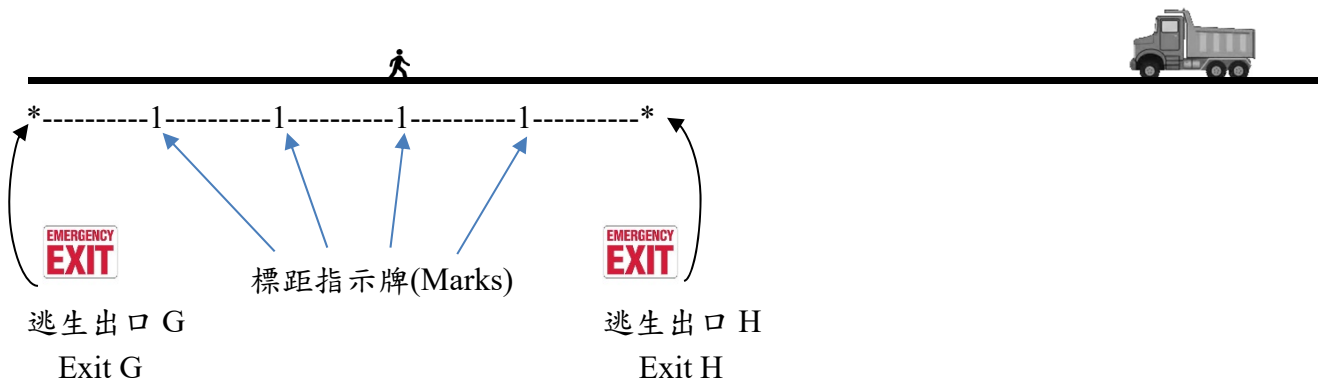


圖 6 / Figure 6

13.

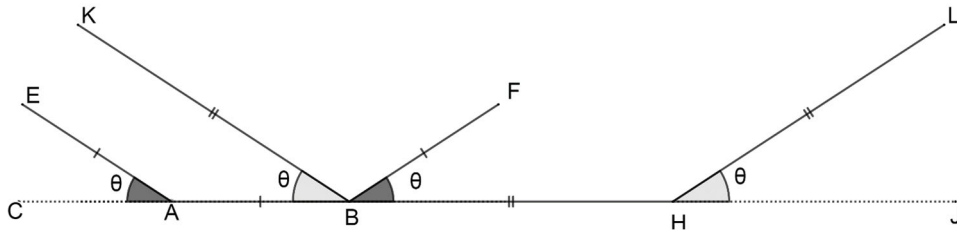


圖 7 / Figure 7

已知 $EA = AB = BF$ 和 $KB = BH = HL$ 使得 $AB : BH = 2 : (1 + \sqrt{5})$ 。

$\angle EAC = \angle KBA = \angle FBH = \angle LHJ = \theta$ 。

Given that $EA = AB = BF$ and $KB = BH = HL$ such that $AB : BH = 2 : (1 + \sqrt{5})$.

$\angle EAC = \angle KBA = \angle FBH = \angle LHJ = \theta$.

(a) KE 延線和 JC 延線相交於 P 點，LF 延線和 JC 延線相交於 Q 點，求 PQ 的長度。

Let P be the intersection of the extends of KE and JC, Q be the intersection of the extends of LF and JC. Find the length of PQ.

(b) 求 $\frac{KE}{FH}$ 之值，準確至三位有效數字。

Find the value of $\frac{KE}{FH}$, correct to 3 significant figures.

14.

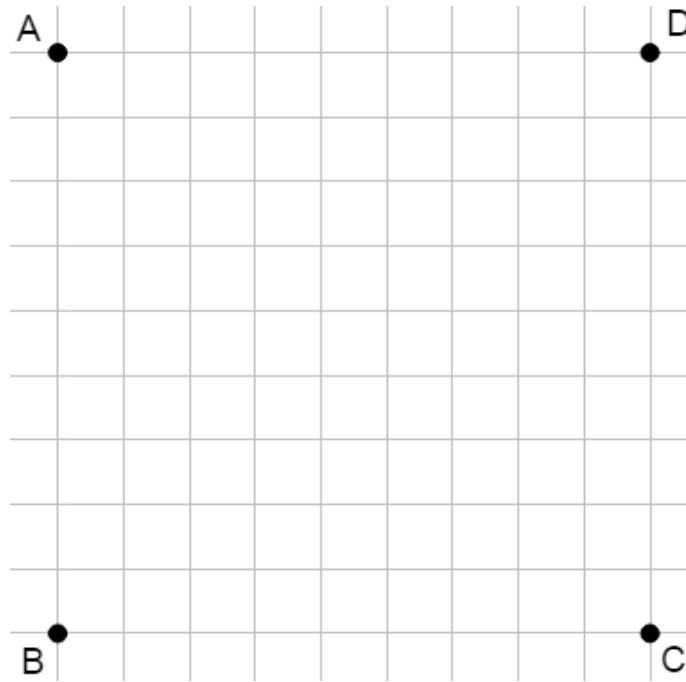
有一八邊形的每條邊長皆為整數，而所有頂點皆在圖中格點上。

A、B、C 和 D 為其中四個頂點。請根據以下的指示畫出八邊形：

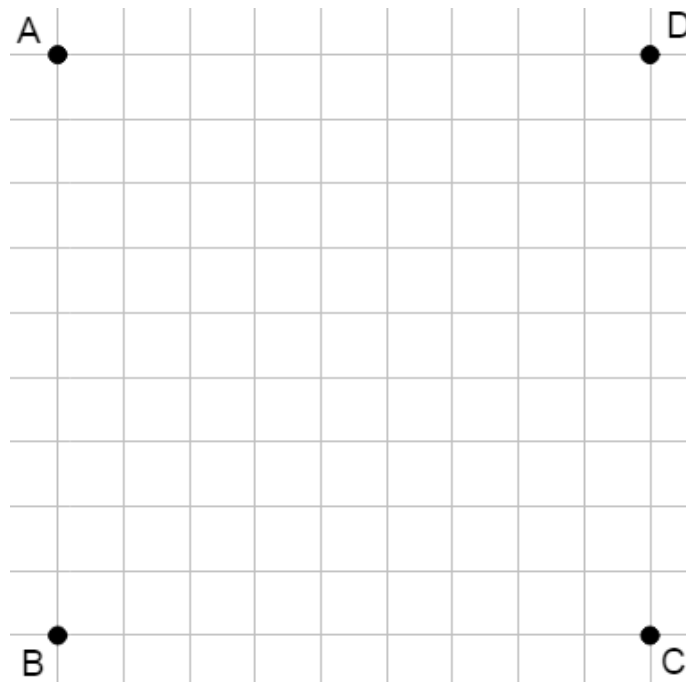
The length of each side of an octagon is integer. All the vertices are located on the intersections of the grid.

A, B, C and D are four of the vertices. Draw the octagons with the following instructions:

(a) 面積最小的八邊形 The octagon with smallest area



(b) 周界最長的八邊形 The octagon with longest perimeter



15.

- (a) 圖 8a 有一單面顏色正方形紙。它可摺成另一正方形 ABCD，使得它有四個全等長方形和一正方形在中央（圖 8b）。

在不用直尺、不撕開/剪開紙張、不繪畫線條的情況下，利用大會提供的正方形紙（附件 A）摺出與圖 8b 條件相同的圖形，但其中中央正方形的面積為紙張面積的 $\frac{1}{16}$ 。

Figure 8a shows a square paper with one side colored. It is folded to form another square ABCD which contains four congruent rectangles and a small square at the center as shown in figure 8b.

Without using ruler, tearing/cutting of paper or drawing any line, use the square paper provided (Appendix A) to fold the figure with the same conditions as figure 8b, but the area of the square at the center is $\frac{1}{16}$ of the area of original paper.

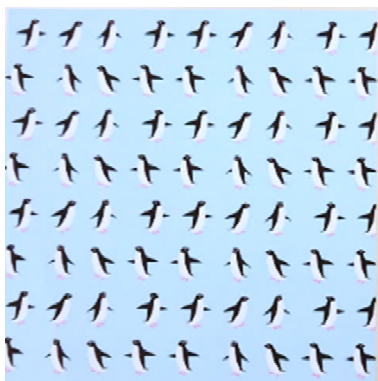


圖 8a/ Figure 8a

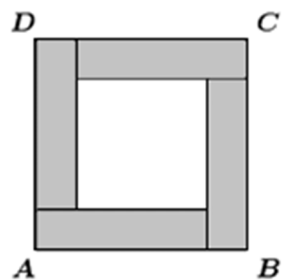


圖 8b/ Figure 8b

(b) 圖 8c 有一單面顏色正方形紙。它可摺成另一正方形 EFHG，使得它有四個全等直角三角形和一正方形在中央（圖 8d）。

在不用直尺、不撕開/剪開紙張、不繪畫線條的情況下，利用大會提供的正方形紙（附件 A）摺出與圖 8d 條件相同的圖形，但其中中央正方形的面積為紙張面積的 $\frac{1}{16}$ 。

Figure 8c shows another square paper with one side colored. It is folded to form another square EFHG which contains four congruent right-angled triangles and a small square at the center as shown in figure 8d.

Without using ruler, tearing/cutting of paper or drawing any line, use the square paper provided (Appendix A) to fold the figure with the same conditions as figure 8d, but the area of the square at the center is $\frac{1}{16}$ of the area of original paper.

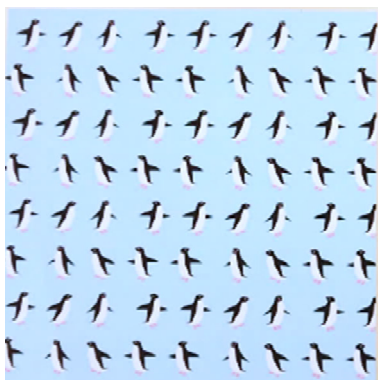


圖 8c/ Figure 8c

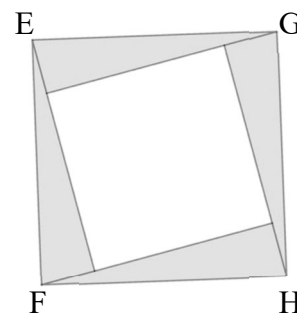


圖 8d/ Figure 8d

(* 把摺好的圖形紙放入文件夾內。)

(* Put all the folded papers into the folder.)

全卷完 [End of Paper]