

2020/21 第十五屆香港小學數學創意解難比賽  
2020/21 The 15<sup>th</sup> Hong Kong Mathematics Creative Problem Solving  
Competition for Primary Schools

題解 Solution

Section A 甲部

1.

符號 ※ 的操作由以下的例子說明。

The symbol ※ is defined by the following examples.

$$1 ※ 2005 = 2006$$

$$2 ※ 2006 = 4014$$

$$3 ※ 2007 = 6024$$

求  $17 ※ 2021$  的值。

Find the value of  $17 ※ 2021$ .

---

**建議答案 Suggested solutions:**

$$1 + 1 \times 2005 = 2006$$

$$2 + 2 \times 2006 = 4014$$

$$3 + 3 \times 2007 = 6024$$

$$17 + 17 \times 2021 = 34374$$

$$17 + 17 \times 2021 = \mathbf{34374}$$

2.

一艘小船以圓形的航道航行。當它航行了 100 米的距離時，它的方向改變了  $30^\circ$ 。求圓形航道的半徑。（答案準確至最接近的米。）

A boat is travelling in a circular path. After it has travelled a distance of 100 m, it has turned  $30^\circ$  in direction. Find the radius of the circular path. (Correct the answer to the nearest meter.)

---

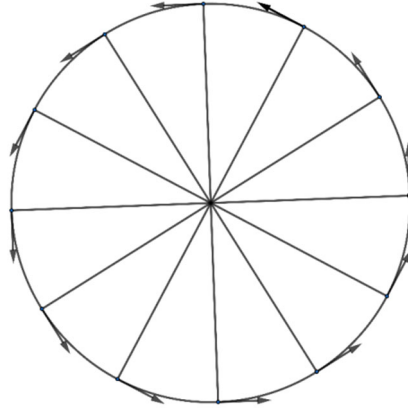
**建議答案 Suggested solutions:**

設半徑為  $r$ 。

Let the radius be  $r$ .

$$2\pi r = 100 \times 12$$

$$r = 191 \text{ m}$$



3.

志明以 3 公里每小時的速度在河上逆流划艇。上午10時，他的袋子掉進水中並隨水飄流。艇前進多 2 公里後，志明發現他丟失了袋子。那一刻，志明把艇掉頭，然後順流划艇，直至他到達袋子的位置。如果志明在靜水中的划艇速度是 7 公里每小時，他在什麼時候才能到達袋子的位置？

Peter was rowing upstream in a river at a speed of 3 km/h . At 10:00 a.m., his bag fell into water and drifted downstream with the current. After rowed 2 km more, he realized that he had lost his bag. At that moment, he turned his boat around and rowed downstream until he reached his bag. If the rowing speed of Peter in still water is 7 km/h, when would he reach his bag?

---

**建議答案 Suggested solutions:**

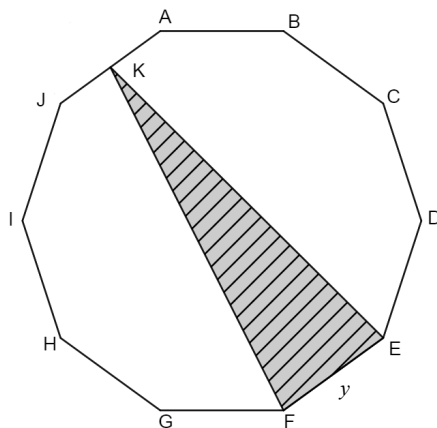
相對水流而言，志明到達袋的位置所需的時間 $=\frac{2}{3} \times 2 = 1\frac{1}{3}$ 小時

志明於 **11:20 a.m.**到達袋的位置。

Relative to the water, the time taken for him to reach his bag after he lost it  $=\frac{2}{3} \times 2 = 1\frac{1}{3}$  hours.

Peter reached his bag at **11:20 a.m.**

4.



圖一 / Figure 1

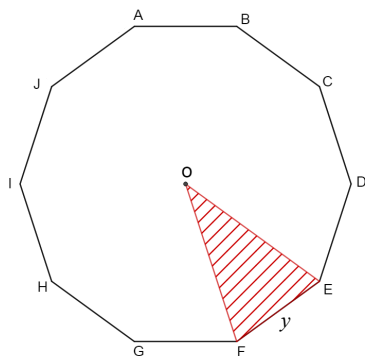
圖一顯示一個邊長為  $y$  的正十邊形。  
求斜線部分面積佔正十邊形面積的幾分之幾。

Figure 1 shows a regular decagon with side length  $y$ .

Write down the fraction of the regular decagon which is shaded.

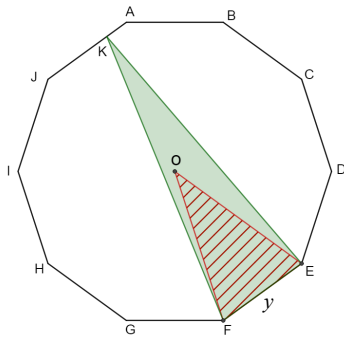
---

**建議答案 Suggested solutions:**



$$\triangle OEF \text{ 面積} = \text{正十邊形面積} \times \frac{1}{10}$$

$$\text{Area of } \triangle OEF = \text{area of regular decagon} \times \frac{1}{10}$$



$\triangle KEF$  和  $\triangle OEF$  有共同的底邊  $EF$ ，但  $\triangle KEF$  的高是  $\triangle OEF$  的兩倍。

$$\triangle KEF \text{ 面積} = \triangle OEF \text{ 面積} \times 2 = \text{正十邊形面積} \times \frac{1}{10} \times 2 = \text{正十邊形面積} \times \frac{1}{5}$$

$\triangle KEF$  and  $\triangle OEF$  have the same base  $EF$  but the height of  $\triangle KEF$  is double of  $\triangle OEF$

$$\text{Area of } \triangle KEF = \text{area of } \triangle OEF \times 2 = \text{area of regular decagon} \times \frac{1}{10} \times 2$$

$$= \text{area of regular decagon} \times \frac{1}{5}$$

5.

書架上擺放着五本不同科目的書，書本的順序符合以下五個條件：

- (I) 歷史書、中文書和科學書是連續擺放的，而歷史書在其餘兩書之間；
- (II) 英文書不是排第一；
- (III) 科學書不是排最後；
- (IV) 數學書與科學書之間相隔了兩本書；
- (V) 英文書不在數學書的旁邊。

寫出五本書擺放的次序。

Five books of different subjects are placed on the bookshelf. The order of the books satisfies the following five conditions:

- (I) The History book, the Chinese book, and the Science book are placed next to each other and the History book is the middle one;
- (II) The English book is not the first;
- (III) The Science book is not the last;
- (IV) Two books are placed between the Mathematics book and the Science book;
- (V) The English book is not next to the Mathematics book.

Write down the order of the five books.

---

**建議答案 Suggested solutions:**

書本 Book	中文 Chinese	英文 English	歷史 History	數學 Mathematics	科學 Science
次序 Order	<b>2</b>	<b>5</b>	<b>3</b>	<b>1</b>	<b>4</b>

6.

把一個圓形蛋糕切成 22 份，每份的大小不一定相等，而每刀都只在蛋糕朝上的面垂直切。問最少要切多少次？

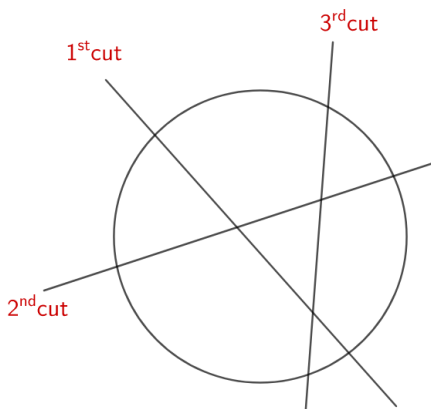
A circular cake is being cut into 22 pieces such that the size of each piece is not necessarily equal and each cut must be perpendicular to the upper surface. What is the minimum number of cuts?

**建議答案 Suggested solutions:**

為確保切最少的次數而得到最多的份數，每次切蛋糕時都要相交所有之前切的線。

To ensure having the maximum number of pieces with minimum cuts, each cut must intersect all the previous cuts.

例/example:



上圖切了 3 刀，第三刀須與第一刀和第二刀的線相交，就能得出最多的份數(7 份)。

The above figure shows 3 cuts. The third cut intersects the first and second cuts to get 7 pieces.

Number of cuts 切的次數	Number of pieces 份數
0	1
1	$1 + 1 = 2$
2	$2 + 2 = 4$
3	$4 + 3 = 7$
4	$7 + 4 = 11$
5	$11 + 5 = 16$
<b>6</b>	$16 + 6 = 22$

7.

甲、乙、丙是三位好朋友，甲欠乙60元，乙欠丙40元，丙欠甲100元，問丙要分別還款多少給甲和乙才能結清欠款？

*A, B, and C are friends. A owes B \$60, B owes C \$40 and C owes A \$100. How much should C repay A and B respectively in order to settle the debts?*

---

**建議答案 Suggested solutions:**

[1M –每個答案 each answer]

A (甲)	B (乙)	C (丙)
60	-60	
	40	-40
-100		100
-40	-20	60

丙應付甲40元及付乙20元。

*C should give A \$40 and should give B \$20 in order to settle the debts.*



8.

在一次家庭聚會上，李太向她的五個孩子每人派發一包糖果，包裝上說明內有 4 至 6 粒糖果。孩子們數算了所得糖果的總數。李先生回到家時，詢問孩子們共有多少粒糖果。所有得到 5 粒糖果的孩子都說了謊話，而其他人都是如實回答。五個孩子的答覆分別是 21、22、23、24 和 25。

求孩子所得糖果的總數。

In a family gathering, Mrs. Lee distributed 5 packs of candies to 5 children. The label on the packing states that the number of candies inside is 4 to 6. The children counted the total number of candies among them. Mr. Lee came home and asked the children how many candies they had among them. All the children with 5 candies lay while others told the truth. The responses of 5 children were 21, 22, 23, 24 and 25 respectively.

Find the total number of the candies among the children.

---

**建議答案 Suggested solutions:**

所有答覆不同  $\Rightarrow$  說真話的孩子總數  $\leq 1$

若果所有孩子都說謊，即真實糖果總數 = 25。  
但這結果與答覆出現的數字「25」存在矛盾。

$\therefore$  即實際情況只有 1 位孩子說真話。

$$\begin{aligned} \therefore \quad & \text{糖果的總數} \\ & = 4 \times 5 + 4 \quad \text{or} \quad 4 \times 5 + 6 \\ & = 24 \quad \text{or} \quad 26 \text{ (rej)} \end{aligned}$$

All the responses are different  $\Rightarrow$  “no. of children telling the truth  $\leq 1$ ”

If all children lie, the actual total candies = 25.

But it contradicts to the response “25”.

$\therefore$  The situation is only 1 child telling the truth.

$$\begin{aligned} \therefore \quad & \text{The total number of candies} \\ & = 4 \times 5 + 4 \quad \text{or} \quad 4 \times 5 + 6 \\ & = 24 \quad \text{or} \quad 26 \text{ (rej)} \end{aligned}$$

9.

假設星球 X 與地球一天的時間長度相同，但表達時間的方式各有不同。星球 X 一天平均分為 8 個♥，每個♥分為 64 個◆。以下為地球時間(24 小時報時制)和星球 X 時間的對照表，求「\*」準確至 1 個◆。

Suppose Planet X has the same time length as the Earth each day, but the methods of description of time are different. Each day on Planet X is evenly divided into 8 ♥. Each ♥ is evenly divided into 64 ◆. The table below shows the comparison between Earth time system (24-hour time) and Planet X time system. Find “\*” correct to the nearest unit ◆.

地球時間/ Earth time	星球 X 時間/ Planet X time
12:00	4♥ 0◆
15:14	5♥ 5◆
20:10	*

---

**建議答案 Suggested solutions:**

地球一天有  $24 \times 60 = 1440$  分鐘

星球 X 一天有  $8 \times 64 = 512$  ◆

地球的 1440 分鐘 = 星球 X 的 512 ◆。

$\therefore$  地球的 1 分鐘 =  $\frac{512}{1440}$  = 星球 X 的 0.35 ◆

由地球時間 12:00 起計算，20:10 過了  $(8 \times 60 + 10) = 490$  分鐘，

即是星球 X 的  $490 \times 0.35 \approx 174$  ◆。

由星球 X 時間 4♥0◆起計算，過了 174 ◆。

$174 \div 64 = 2 \dots 46$

即是過了 2♥46◆。

$\therefore$  地球時間 20:10 = 星球 X 時間 6♥46◆。

The Earth has  $24 \times 60 = 1440$  min per day.

Planet X has  $8 \times 64 = 512$  ◆ per day.

1440 minutes on Earth = 512 ◆ on Planet X

$\therefore$  1 min on Earth =  $\frac{512}{1440}$  = 0.35 ◆ on Planet X

Count from Earth time 12:00, 20:10 has passed through  $(8 \times 60 + 10) = 490$  min. That equals to 490

$\times 0.35 \approx 174$  ◆ on Planet X.

Count from Planet X time 4♥0◆, 174 ◆ has passed.

$174 \div 64 = 2 \dots 46$

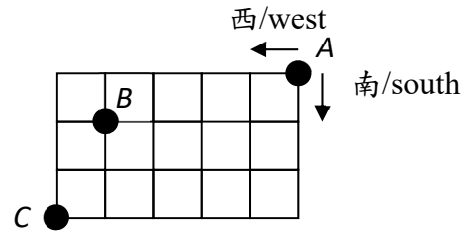
2♥ and 46♦ has passed.

∴ Earth time 20:10 = Planet X time 6♥46♦

10.

圖二顯示一城市網絡圖。小明現位於  $A$  點。假定小明只可向南面或西面前進。求小明由  $A$  點步行至  $C$  點但不經過  $B$  點的路線數目。

Figure 2 shows a network of a city. John is now at  $A$ . Suppose he can move towards south or west only. Find the number of different ways for him to walk from  $A$  to  $C$  **without passing  $B$** .



圖二 / Figure 2

**建議答案 Suggested solutions:**

下圖顯示由  $A$  至  $C$  的路線數目：

The following shows the number of ways to walk from  $A$  to  $C$ :

	1	1	1	1	1	$A$
	6	5	4	3	2	1
	21	15	10	6	3	1
$C$	56	35	20	10	4	1

下圖顯示由  $B$  至  $C$  的路線數目：

The following shows the number of ways to walk from  $B$  to  $C$ :

	1	$B$		
	2	1		
$C$	3	1		

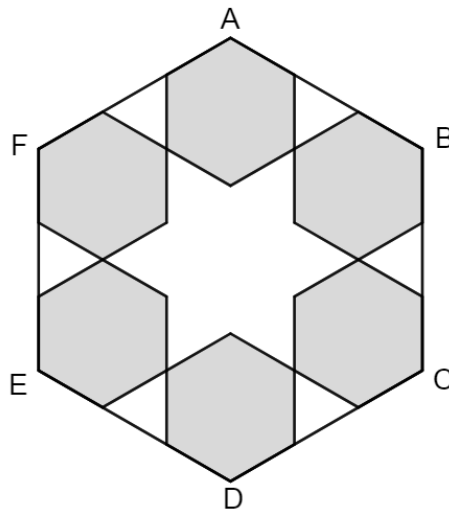
因此，由  $A$  至  $C$  且通過  $B$  的路線數目  $= 5 \times 3 = 15$

Therefore, the number of ways to walk from  $A$  through  $B$  to  $C = 5 \times 3 = 15$

因此，由  $A$  至  $C$  且不通過  $B$  的路線數目  $56 - 15 = 41$ 。

Hence, the number of ways to walk from  $A$  to  $C$  without passing  $B = 56 - 15 = 41$ .

11.



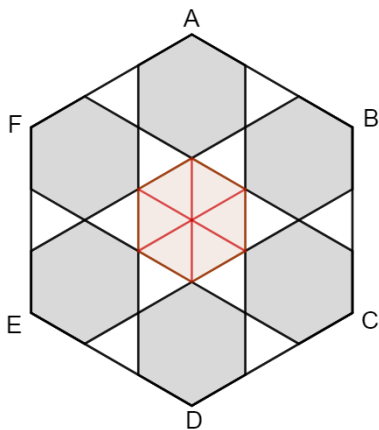
圖三/ Figure 3

圖三的陰影部分為六個完全相同的小正六邊形，陰影部分的總面積是  $24 \text{ cm}^2$ 。求六邊形 ABCDEF 的面積。

The shaded parts in Figure 3 are 6 identical small regular hexagons and the total area of the shaded parts is  $24 \text{ cm}^2$ . Find the area of hexagon ABCDEF.

---

建議答案 Suggested solutions:



每個小正六邊形可分割成 6 個小等邊三角形。

全圖有  $6 \times 6 + 6 + 12 = 54$  個小等邊三角形。

六邊形 ABCDEF 的面積 =  $24 \div (6 \times 6) \times 54 = 36 \text{ cm}^2$

Each small regular hexagon can be divided into 6 small equilateral triangles.

There are  $6 \times 6 + 6 + 12 = 54$  small equilateral triangles in the figure.

Area of hexagon ABCDEF =  $24 \div (6 \times 6) \times 54 = 36 \text{ cm}^2$

12.

使用四則運算符號、括號及所有 1 至 10 的整數，每個數字必須及只能出現一次，寫出答案為 2021 的算式，當中數字無須順序。

Using the four main arithmetic operations, parentheses and all integers from 1 to 10, each integer can be used once and only once, write down an expression with result 2021. The numbers need not be in order.

求得 39 的算式例子：

Example of expression with result 39:

$$(6 + 5) \times 4 - 10 + 9 \div 3 \times 2 \times (8 - 7) - 1 = 39$$

---

**建議答案 Suggested solutions:**

$$4 \times 5 \times (9 \times 6 + 8 \times 7 - 10 + 3 - 2) + 1 = 2021$$

or

$$(8 \times (3 + 5) + 6 \times 7 + 4 - 9) \times 2 \times 10 + 1 = 2021$$

接受其他合理答案。

Accept other possible answers.

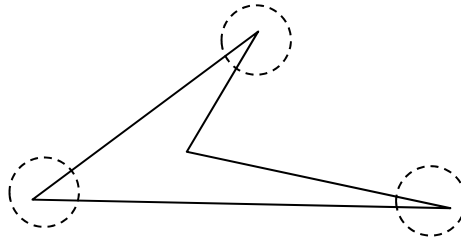
13.

在答題紙提供的方格中，清晰畫出一個有最多內角為銳角的七邊形，並圈起所有銳角的內角。

In the graph paper provided in the answer sheet, draw a heptagon with the maximum number of acute interior angles clearly and circle all the interior angles that are acute.

例如下圖顯示一個四邊形，並圈起三隻銳角的內角。

For example, the following figure shows a quadrilateral with 3 acute interior angles circled.



**建議答案 Suggested solutions:**

[1M – 畫圖 / 1M – 圈出正確的銳角]

七邊形的內角總和 =  $(7-2) \times 180^\circ = 900^\circ$

如果所有內角都是銳角，內角總和  $< 7 \times 90^\circ = 630^\circ$  (捨去)

如果6個內角都是銳角，其中6個內角總和  $< 6 \times 90^\circ = 540^\circ$ ，餘下的1個內角  $> 360^\circ$  (捨去)

如果5個內角都是銳角，其中5個內角總和  $< 5 \times 90^\circ = 450^\circ$ ，餘下的2個內角總和  $> 450^\circ$ ，即每個角平均  $> 225^\circ$  (接受)

$\therefore$  七邊形的內角最多有 **5個銳角**。

[1M – drawing / 1M – circle the right acute angles]

Sum of interior angles of heptagon =  $(7-2) \times 180^\circ = 900^\circ$

If all the interior angles are acute, sum of interior angles  $< 7 \times 90^\circ = 630^\circ$  (**rejected**)

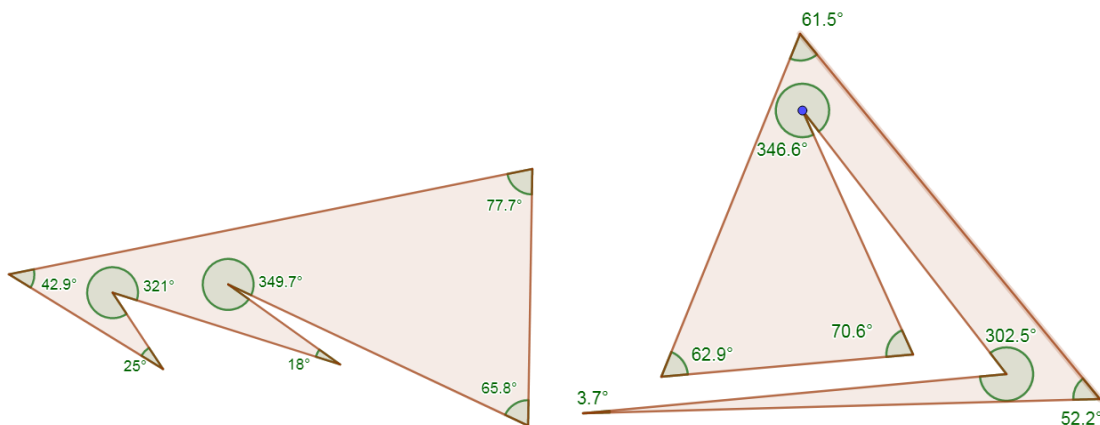
If 6 interior angles are acute, sum of that 6 interior angles  $< 6 \times 90^\circ = 540^\circ$ ，and the remaining interior angle  $> 360^\circ$  (**rejected**)

If 5 interior angles are acute, sum of that 5 interior angles  $< 5 \times 90^\circ = 450^\circ$ ，and the sum of remaining 2 interior angles  $> 450^\circ$ ，which means one interior angle  $> 225^\circ$  in average (**accepted**)

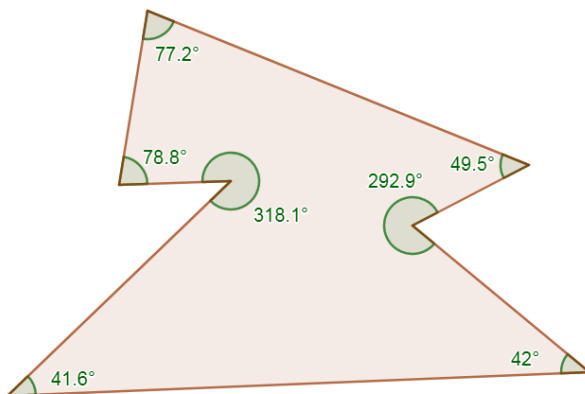
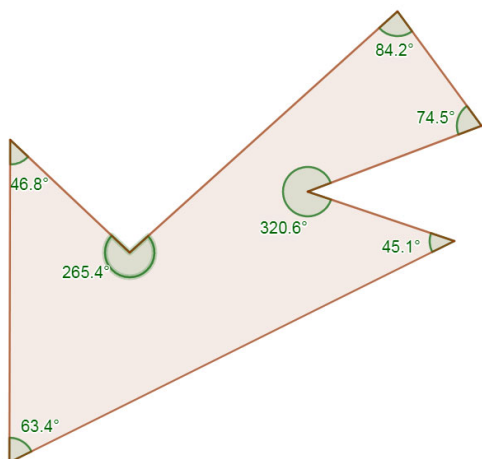
$\therefore$  The maximum number of acute interior angles of a heptagon is 5.

有5個內角是銳角的七邊形例子:

Examples of a heptagon with 5 acute interior angles:







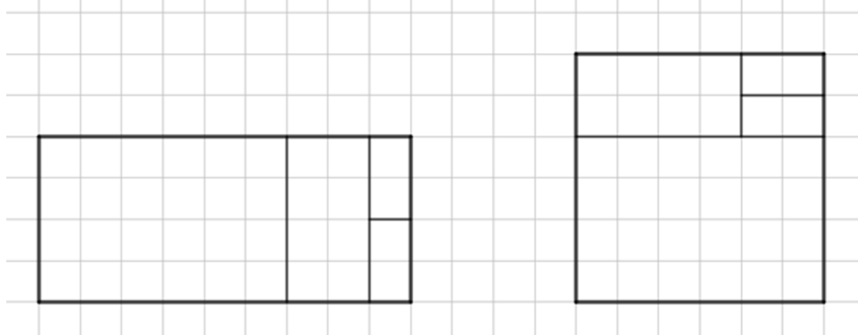
接受其他合理答案。

Accept other possible answers.

14.

我們可以把長方形加上直線，把它分割成數份，並重新組合成一個正方形。  
圖四為一個分割的例子。

We can draw straight lines on the rectangle to cut it into several pieces, and rearrange the pieces to form a square. Figure 4 is an example.



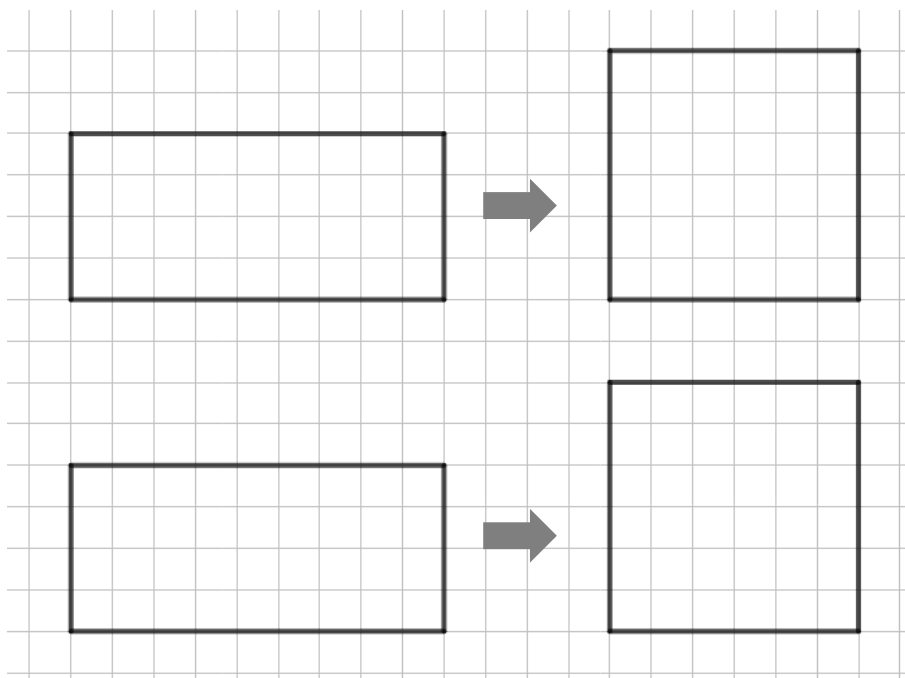
圖四/ Figure 4

若兩種切割方法得出來的圖形組互為全等，則此兩種方法視之為相同的切割方法。

If the set of pieces obtained by two cutting methods are identical, we say these two cutting methods are the same.

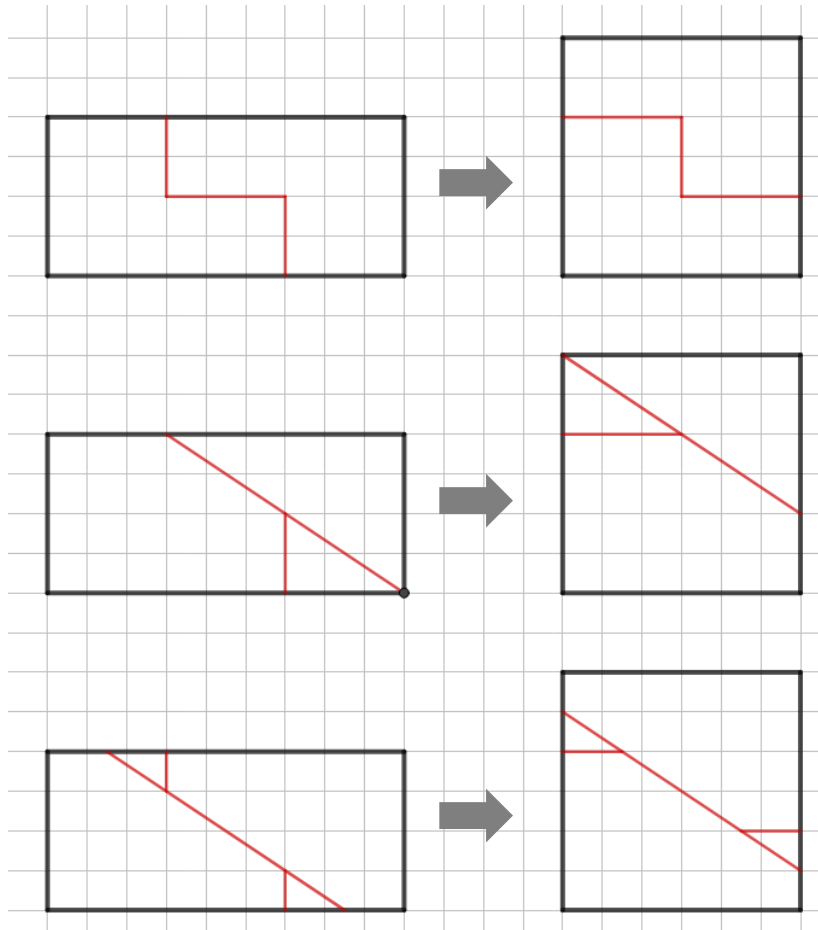
試設計兩種與例子不同的分割方法，把下圖的長方形加畫直線，分割成少於 5 塊，重新組合成一個正方形，並在正方形上畫直線顯示如何組合。

Design two cutting methods different from the example. Draw straight lines on each of the rectangles below to cut it into less than 5 pieces. Rearrange the pieces to form a square and draw straight lines on the squares to show the combinations.



建議答案 Suggested solutions:

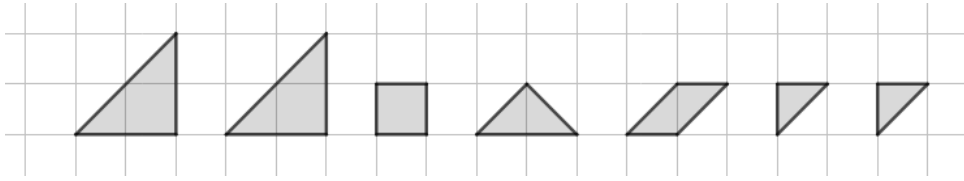
(1M – 每個方法 /each method)



接受其他合理答案。

Accept other possible answers.

15.



圖五/ Figure 5

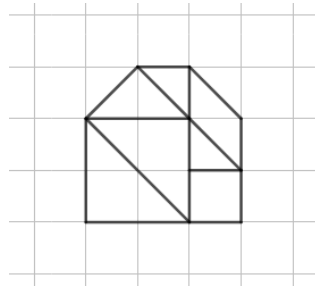
利用圖五顯示的七巧板拼出：

Use tangram as shown in Figure 5 to make:

- (a) 能夠密鋪的五邊形;  
a pentagon that can tessellate;
- (b) 能夠密鋪的六邊形。  
a hexagon that can tessellate.

把圖形在方格圖中畫出來。下圖顯示了**不能夠密鋪**的六邊形例子。

Draw the figures on the grids. The figure below shows an example of a hexagon that **cannot tessellate**.



---

**建議答案 Suggested solutions:**

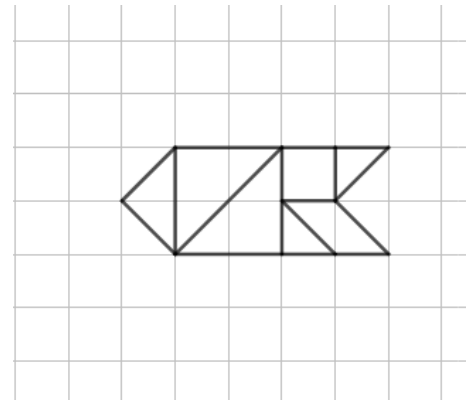
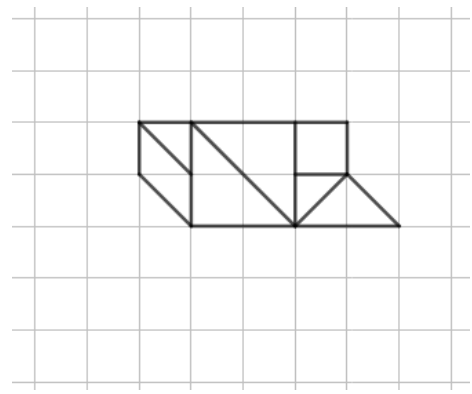
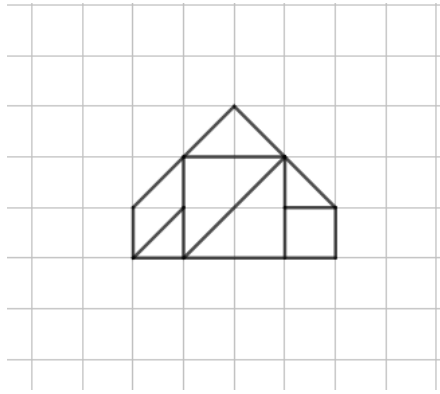
(1M – 每個圖形 /each figure)

(a) 能夠密鋪的五邊形

A pentagon that can tessellate

(b) 能夠密鋪的六邊形

A hexagon that can tessellate



接受其他合理答案。

Accept other possible answers.

## Section B 乙部

以下是 A 國有關男性和女性的每年平均死亡年齡統計。

The following is the annual average age of death statistics of men and women in Country A.

年份	當年平均死亡年齡	
	男性 Men	女性 Women
2001	61.2	70.6
2002	61.3	70.5
2003	61.3	70.4
2004	61.8	70.8
2005	61.6	70.6
2006	62.2	71.5
2007	62.2	71.4
2008	62.2	71.5
2009	62.6	71.9
2010	62.9	72
2011	63.1	72.7
2012	63.5	72.4
2013	63.9	72.7
2014	64	72.9
2015	64.2	73.3
2016	64.1	73.3
2017	64.7	73.6
2018	65.1	73.7
2019	65	74.1
2020	65.5	74.1

- (a) 假設你全校的同學都於 A 國出生並生活，試以 A 國的數據估算和你同齡的女同學和男同學的平均預期壽命。

Assuming that all your schoolmates were born and live in Country A, estimate the expected average life of female schoolmates and male schoolmates of the same age as yours based on the statistics of Country A.

- (b) A 國的數據顯示女士的平均預期壽命一向比男士長。參考以上數據，你認為將來會否可能出現男士的平均壽命較女士長？為什麼？

The statistics of Country A shows that expected average life of women has always been longer than that of men. With reference to the above data, do you think it is possible that men's expected average life will be longer than women's in the future? Why?

**建議答案 Suggested solutions:**

(a)

假設作答者的年齡是 11 歲。

在過去的 20 年中，男性和女性預期壽命的每年平均增幅為：

男性：每年  $\frac{65.5-61.2}{20} = 0.215$  歲

女性：每年  $\frac{74.1-70.6}{20} = 0.175$  歲

Assume the age of the kid is 11.

In the past 20 years, the average increments of expected life span per year for men & women are:

$$\frac{65.5-61.2}{20} = 0.215 \text{ per year for men}$$

$$\frac{74.1-70.6}{20} = 0.175 \text{ per year for women}$$

男性和女性 70 年後的預期壽命是：

The expected life span for 70 years later for men and women are:

$$65.5 + 70 \times 0.215 = 80.55 \text{ (男/men)}$$

$$74.1 + 70 \times 0.175 = 86.35 \text{ (女/women)}$$

**更嚴謹的處理手法：**

**For more rigorous treatment:**

$$65.5 + 0.215n = 11 + n$$

$$n = 69.4$$

男性的預期壽命是：

The expected life span for men is:

$$11 + 69.4 = 80.4$$

$$74.1 + 0.175n = 11 + n$$

$$n = 76.5$$

女性的預期壽命是：

The expected life span for women is  $11 + 76.5 = 87.5$

以下是採用相同方法得出的其他預期壽命，以供參考

Some other expected life span by the same approach are listed below for reference

作答者的年齡 Age of kid	9	10	12	13	14
男性預期壽命 Expected life of men	81.0	80.7	80.2	79.9	79.6
女性預期壽命 Expected life of women	87.9	87.7	87.3	87.1	86.8

(b)

答案可以是「是」或「否」。

「否」的原因之一是這裡只是使用了線性模型。

Can be both “yes” or “no”.

One reason for “no” is that the model we used here is only linear.